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PRACTICAL DESIGN ISSUES OF TUNED MASS DAMPERS FOR TORSIONALLY COUPLED BUILDINGS UNDER EARTHQUAKE LOADINGS

JIN-MIN UENG[1], CHI-CHANG LIN[2]* AND JER-FU WANG[1]
[1] *Department of Civil Engineering, National Chung Hsing University, Taichung, Taiwan, ROC*
[2] *College of Engineering, and Department of Civil Engineering, National Chung Hsing University, Taichung, Taiwan, ROC*

SUMMARY

In this study, a new design procedure was developed for reducing the dynamic responses of torsionally coupled buildings, particularly existing buildings, under bilateral earthquake excitations, by incorporating the vibration control effectiveness of passive tuned mass dampers (PTMDs) . Some practical design issues such as the optimal location for installation, movement direction and numbers of PTMD are considered in this study. The optimal parameters of the PTMD system are determined by minimizing the mean square displacement response ratio of the controlled degree of freedom between the building with and without PTMDs. In addition, parametric studies of the PTMD planar position and the detuning effect are undertaken to determine their influence on the response control efficacy. The numerical results from two typical multistory torsionally coupled buildings under bidirectional ground accelerations, recorded at the 1979 El Centro earthquake, verify that the proposed optimal PTMDs are more effective and more robust in reducing the building responses. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

Over the past 20 years, several innovative passive and active control devices have been developed for reducing vibrations in human-made structures caused by wind loads and earthquakes. Tuned mass damper (TMD) systems, since their installation in the CN Tower in Toronto in 1975 and in the John Hancock Building in Boston in 1977, have been one of the major vibration control devices for civil engineering structures. TMDs, which consist of a mass, a spring and a viscous damper attached to the structure, have been used to reduce the dynamic responses of structures under strong environmental loads (wind and earthquakes). The mechanism for mitigating structural vibration using a TMD involves transferring the vibration energy from the structure into the TMD, which dissipates this energy through the damping effect.

Through intensive research and development in recent years, the passive tuned mass damper (PTMD) has been accepted as an effective vibration control device for both new and existing structures, to enhance their reliability against winds, earthquakes, and human activities (Wirsching and Campbell, 1974; McNamara, 1977; Luft, 1979; Kwok, 1984; Rainer and Swallow, 1986; Kwok and Macdonald, 1990; Thornton *et al.*, 1990; Satareh and Hanson, 1992; Xu *et al.*, 1992; Kawaguchi *et al.*, 1992; Villaverde and Koyama, 1993; Sinha and Igusa, 1995). Determining the optimal PTMD

*Correspondence to: Chi-Chang Lin, Department of Civil Engineering, National Chung Hsing University, Taichung, Taiwan 40227, ROC. E-Mail: cclin3@dragon.nchu.edu.tw

system parameters (i.e. the mass, damping and stiffness coefficients) to effectively decrease structural vibrations induced by different types of excitations is now well established (Warburton, 1982; Asami *et al.*, 1991; Fujino and Abe, 1993; Lin *et al.*, 1994; Xu and Kwok, 1994). It is well recognized that the performance of a PTMD is sensitive to the frequency ratio between the PTMD and the structure. Even a slight deviation in the frequency ratio from its design value, either due to a drift in the PTMD frequency or the structural frequency, called the detuning effect, would result in a drastic deterioration in the control effectiveness of the PTMD. In more recent studies (Abe and Fujino, 1994; Igusa and Xu, 1994; Jangid, 1995, 1999; Abe and Igusa, 1995; Ram and Elhay, 1996; Li, 2000, 2002; Park and Reed, 2001; Wang and Lin, 2005; Lin *et al.*, 2005), multiple tuned mass dampers (MTMDs) with distributed natural frequencies near the fundamental frequency of the main structure were proposed to reduce the detuning effect. Most of the previous studies considered the controlled structure as a single degree-of-freedom (SDOF) system with its fundamental modal properties in order to simplify the optimization problem when designing the PTMD and MTMD. Much of the research in this area has been done with the aim of controlling only a single mode.

However, in reality, a building generally possesses a large number of degrees of freedom, and is actually asymmetrical, even with a nominally symmetrical plan. Such a structure will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. An earlier review shows that considerable work was done on the PTMD and the MTMD to reduce the seismic response of buildings idealized as a simplified SDOF or planar model. Few researchers employed an asymmetrical structural model for investigating the response control effectiveness of PTMDs and MTMDs. Jangid and Dutta (1997) studied the response control of a 2-DOF torsional system using a cluster of MTMDs. Lin and co-workers (Ueng and Lin, 1996; Ueng *et al.*, 1998; Lin *et al.*, 2000) investigated a multistory torsional building system with one and two TMDs. Singh *et al.* (2002) used four TMDs, placed along two orthogonal directions in pairs, to control the coupled lateral and torsional response of a multistory building. This study showed that the simplified SDOF or planar model, which ignores the structural lateral–torsional coupling and the PTMD effect on different modes, could overestimate the PTMD control effectiveness.

It is well known that vibration control in building structures using a PTMD is mainly attributed to the suppression of the controlled modal responses. For a torsionally coupled building, the first three modes dominate the translational and torsional floor responses. Because floor translations in the principal directions have different dominant modes, it is possible to reduce the dynamic responses for all degrees of freedom using one PTMD. However, the single PTMD does not provide the same optimal effectiveness to every DOF. Therefore, the MTMD concept, having a separate PTMD for every structural dominant mode, appears to be worth investigating. Rana and Soong (1998) investigated such an MTMD for controlling multiple structural modes of a planar building structure. Therefore, it is of interest to study the response control for torsional buildings using several PTMDs. Since the PTMD is constrained to move in one direction only, it is usually most effective when the structural response is predominant in the direction that the damper is designed to move. However, because the direction in which the earthquake 'hits' a structure is not known *a priori*, a fixed damper configuration may very well have a reduced control performance. Lin and co-workers (Ueng and Line, 1996; Ueng *et al.*, 1998; Lin *et al.*, 2000) found that the vibration control effectiveness of a PTMD depends not only on the controlled modal parameters of the primary structure, but also the location it is installed in, and the direction of the PTMD movement as well as the earthquake direction. For a torsionally coupled structure under bidirectional earthquake excitations, using the simplified model and ignoring the location of the installation and the direction of the movement of the PTMD could lead to incorrect PTMD design and an overestimation of vibration control required.

The primary objective of this paper is to investigate the vibration control effectiveness of PTMDs for torsionally coupled buildings under bidirectional earthquake excitations. Some practical considerations such as the optimal installed floor, planar position and PTMD movement direction are investigated first. The optimal PTMD system parameters are then determined by minimizing the mean square displacement response ratio of the top floor between the building with and without the PTMD. In addition, parametric studies regarding the PTMD planar position and detuning effect, due to the variations of PTMD frequency and damping ratio, were investigated to determine their influence on the response control efficacy. Numerical results from several typical multistory torsionally coupled buildings, subjected to the bidirectional ground accelerations of the 1979 El Centro earthquake, verify that the proposed optimal PTMDs are able to effectively reduce the building responses.

The PTMD design method developed in this study possesses the following characteristics: (1) the building is modeled as a torsionally coupled structure, and the PTMD's optimum installation location both in plan and in elevation is considered; the direction of the PTMD movement differs from that presented in most papers where the building is considered as a SDOF or MDOF planar system; (2) the mean square response of a specified degree of freedom DOF is minimized by considering all of the important modes, while most of the papers presented to date minimize only one controlled modal response; (3) only the first few important (or identified) modal parameters are needed. Again, this differs from most papers (Jangid and Datta, 1997; Singh *et al.*, 2002), which required the knowledge of the system matrices in order to design the PTMDs.

2. DYNAMIC EQUATION OF BUILDING–PTMD SYSTEMS

In a general torsionally coupled N-story shear building, as shown in Figure 1, each floor has 3 degrees of freedom: x-displacement, y-displacement, and rotation about the vertical axis relative to the ground at the center of mass. On floor l, they are denoted by x_l, y_l and θ_l, respectively. Assuming an SDOF PTMD with mass m_{sy}, damping coefficient c_{sy} and stiffness k_{sy}, installed at the lth floor at a distance of d_{sy} to the y-axis of the lth floor moving in the y direction, as shown in Figure 2, then the dynamic equation of motion for the combined building–PTMD system under an oblique incident, horizontal earthquake excitation with incident angle β from the x-axis can be written as

$$\begin{bmatrix} \boldsymbol{M_p} & 0_{3N\times 1} \\ 0_{1\times 3N} & m_{sy} \end{bmatrix}\begin{Bmatrix} \ddot{\boldsymbol{u}}_{\boldsymbol{p}} \\ \ddot{u}_{sy} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C_p} & 0_{3N\times 1} \\ 0_{1\times 3N} & c_{sy} \end{bmatrix}\begin{Bmatrix} \dot{\boldsymbol{u}}_{\boldsymbol{p}} \\ \dot{u}_{sy} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K_p} & 0_{3N\times 1} \\ 0_{1\times 3N} & k_{sy} \end{bmatrix}\begin{Bmatrix} \boldsymbol{u_p} \\ u_{sy} \end{Bmatrix}$$
$$+\begin{Bmatrix} 0 \\ \vdots \\ 0 \\ (k_{sy}[-u_{sy}+y_l+d_{sy}\theta_l])_{(3l-1)th\ row} \\ (d_{sy}k_{sy}[-u_{sy}+y_l+d_{sy}\theta_l])_{(3l)th\ row} \\ 0 \\ \vdots \\ 0 \\ (k_{sy}[-y_l-d_{sy}\theta_l])_{(3N+1)th\ row} \end{Bmatrix} + \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ (c_{sy}[-\dot{u}_{sy}+\dot{y}_l+d_{sy}\dot{\theta}_l])_{(3l-1)th\ row} \\ (d_{sy}c_{sy}[-\dot{u}_{sy}+\dot{y}_l+d_{sy}\dot{\theta}_l])_{(3l)th\ row} \\ 0 \\ \vdots \\ 0 \\ (c_{sy}[-\dot{y}_l-d_{sy}\dot{\theta}_l])_{(3N+1)th\ row} \end{Bmatrix} \tag{1}$$
$$= -\begin{bmatrix} \boldsymbol{M_p} & 0_{3N\times 1} \\ 0_{1\times 3N} & m_{sy} \end{bmatrix}\begin{Bmatrix} \boldsymbol{r}_{3N\times 1} \\ (\sin\beta)_{(3N+1)th\ row} \end{Bmatrix}\ddot{u}_g$$

In Equation (1), $\boldsymbol{M}_p$, $\boldsymbol{C}_p$ and $\boldsymbol{K}_p$ are the $3N \times 3N$ mass, damping and stiffness matrices, respectively, of the primary building; u_{sy} denotes the PTMD displacement relative to the ground. Assuming that $\boldsymbol{C}_p$

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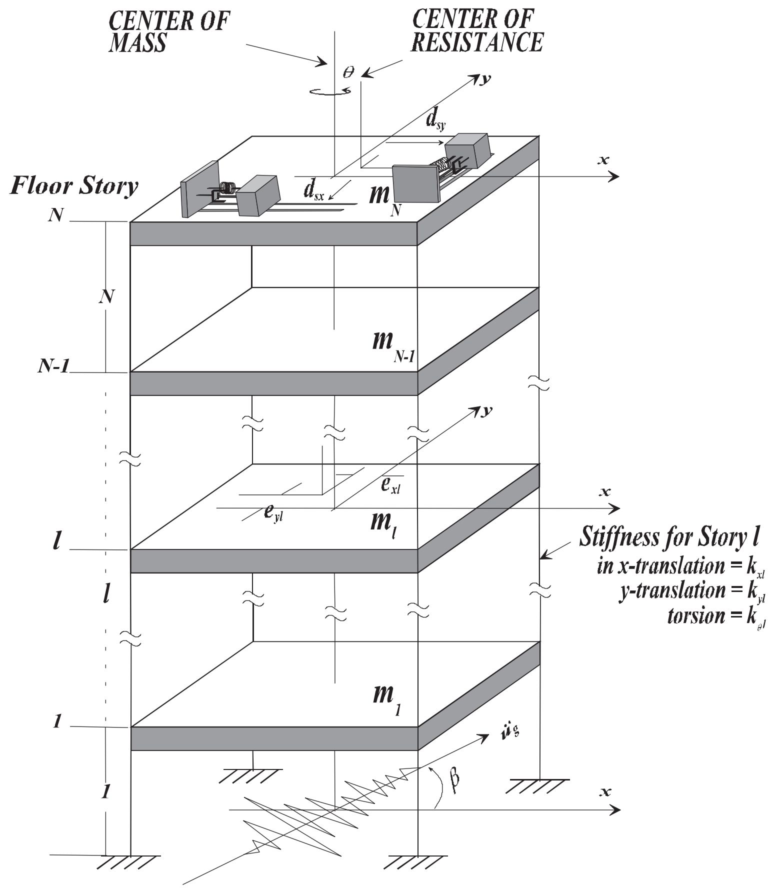

Figure 1. *N*-story general torsionally coupled building–PTMD system

is a classical damping matrix and defining $\mu_{jy} = (\phi_{3l-1,j} + d_{sy}\phi_{3l,j})m_{sy}/m_j^*$, the equation of motion for jth mode of the controlled structure is expressed as

$$\ddot{\eta}_j + 2\xi_j\omega_j\dot{\eta}_j + \omega_j^2\eta_j - \mu_{jy}(2\xi_{sy}\omega_{sy}\dot{v}_{sy} + \omega_{sy}^2 v_{sy}) = -\frac{1}{m_j^*}(\boldsymbol{\phi}_j^T \boldsymbol{M}_p \boldsymbol{r})\ddot{u}_g \tag{2}$$

In Equation (2), $v_{sy} = u_{sy} - (y_l + d_{sy}\theta_l)$ is the PTMD displacement relative to the lth floor, or the PTMD stroke; m_j^* and η_j are the jth generalized modal mass and displacement. $\omega_{sy} = \sqrt{k_{sy}/m_{sy}}$ and $\xi_{sy} = c_{sy}/(2m_{sy}\omega_{sy})$ represent the natural frequency and damping ratio of the PTMD, respectively. $\phi_{3l-1,j}$ denotes the $(3l-1)$th element of the jth mode shape ϕ_j. Similarly, the equation of motion for the PTMD in Equation (1) becomes

$$\left(\sum_{j=1}^{3N}(\phi_{3l-1,j} + d_{sy}\phi_{3l,j})\ddot{\eta}_j\right) + \ddot{v}_{sy} + 2\xi_{sy}\omega_{sy}\dot{v}_{sy} + \omega_{sy}^2 v_{sy} = -\sin\beta\cdot\ddot{u}_g \tag{3}$$

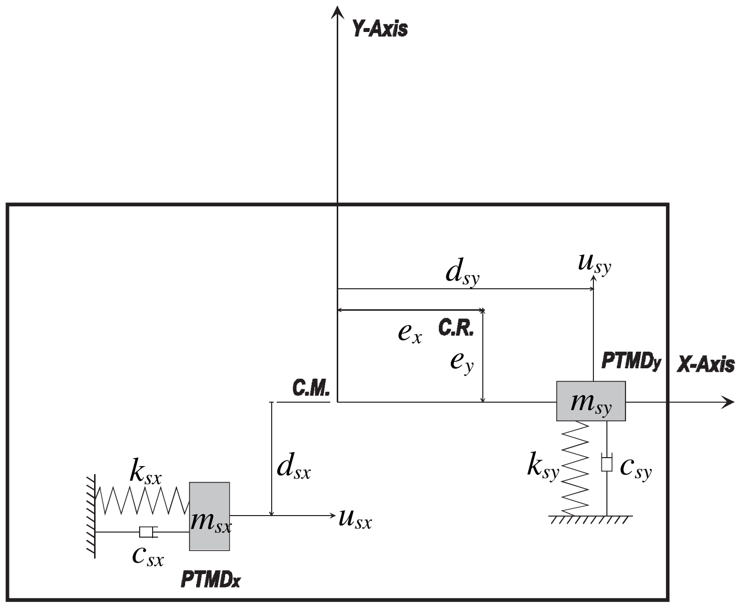

Figure 2. Planar position of two PTMDs

Provided the first three modes make the greatest contribution to the structural responses from Equations (2) and (3), the equations of motion for the first three modes of the primary structure and PTMD are expressed in matrix form as

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \phi_{3l-1,1}+d_{sy}\phi_{3l,1} & \phi_{3l-1,2}+d_{sy}\phi_{3l,2} & \phi_{3l-1,3}+d_{sy}\phi_{3l,3} & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \\ \ddot{\eta}_3 \\ \ddot{v}_{sy} \end{Bmatrix}
+ \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 & -2\mu_{1y}\xi_{sy}\omega_{sy} \\ 0 & 2\xi_2\omega_2 & 0 & -2\mu_{2y}\xi_{sy}\omega_{sy} \\ 0 & 0 & 2\xi_3\omega_3 & -2\mu_{3y}\xi_{sy}\omega_{sy} \\ 0 & 0 & 0 & 2\xi_{sy}\omega_{sy} \end{bmatrix} \begin{Bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \\ \dot{v}_{sy} \end{Bmatrix}
+ \begin{bmatrix} \omega_1^2 & 0 & 0 & -\mu_{1y}\omega_{sy}^2 \\ 0 & \omega_2^2 & 0 & -\mu_{2y}\omega_{sy}^2 \\ 0 & 0 & \omega_3^2 & -\mu_{3y}\omega_{sy}^2 \\ 0 & 0 & 0 & \omega_{sy}^2 \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ v_{sy} \end{Bmatrix}
= \begin{Bmatrix} -\dfrac{1}{m_1^*}(\phi_1^T \boldsymbol{M}_p \boldsymbol{r}) \\ -\dfrac{1}{m_2^*}(\phi_2^T \boldsymbol{M}_p \boldsymbol{r}) \\ -\dfrac{1}{m_3^*}(\phi_3^T \boldsymbol{M}_p \boldsymbol{r}) \\ -\sin\beta \end{Bmatrix} \ddot{u}_g \tag{4}
$$

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In the case of the primary structure without PTMD, the jth modal equation of motion is given by

$$\ddot{\eta}_j + 2\xi_j\omega_j\dot{\eta}_j + \omega_j^2\eta_j = -\frac{1}{m_j^*}(\phi_j^T \boldsymbol{Mr})\ddot{u}_g \tag{5}$$

Based on Equations (4) and (5), the responses between the building with and without PTMD can be compared to determine the optimal PTMD system parameters.

3. OPTIMAL SYSTEM PARAMETERS OF ONE PTMD

In this paper, the optimal PTMD system parameters are determined by minimizing the mean square displacement response ratio of the ith DOF, $R_{dE,i}$, between a building with and without a PTMD under earthquake excitation. Assuming $\ddot{u}_g(t)$ is a stationary random process with power spectral density $S_{\ddot{u}_g}(\omega)$, then $R_{dE,i}$ takes the following form:

$$\boldsymbol{R}_{dE,i} = \frac{E[u_i^2]_{\text{One PTMD}}}{E[u_i^2]_{\text{No PTMD}}};\ E[u_i^2] = \int_{-\infty}^{\infty}\left|\sum_{j=1}^{3N}\phi_{i,j}H_{\eta_j\ddot{u}_g}(\omega)\right|^2 S_{\ddot{u}_g}(\omega)\mathrm{d}\omega \tag{6}$$

where $H_{\eta_j\ddot{u}_g}(\omega)$ is the transfer function of the jth modal displacement with respect to $\ddot{u}_g(t)$. The transfer functions of the primary structure with and without a PTMD can be obtained by taking the Fourier transform of Equations (4) and (5), respectively. Lin *et al.* (1994) concluded that the PTMD is useful for flexible structures founded on firm ground. This indicates that the dominant frequency of excitation is higher than that of the structure. Thus, without loss of accuracy, it is assumed that the earthquake excitation is a white-noise random process, i.e. $S_{\ddot{u}_g}(\omega) = S_0$. An $R_{dE,i}$ value smaller than unity represents the reduction of the structural response due to the presence of the PTMD.

For a torsionally coupled building, the displacement response at each DOF is contributed to mainly by the first three modes. Therefore, from Equation (6), $R_{dE,i}$ is a function of the building modal properties (ω_j, ξ_j, ϕ_j; j = 1, 2 or 3) and the installed floor ($\phi_{3l-1,j}$, $\phi_{3l,j}$), the movement direction and planar position of the PTMD (d_{sy}), and the PTMD parameters (m_{sy}, ω_{sy}, ξ_{sy}). These factors play very important roles in the optimum PTMD design and control its efficacy. With the building modal parameters, the installed floor and PTMD movement direction known, the optimal PTMD parameters can be obtained by solving the following equations:

$$\frac{\partial R_{dE,i}}{\partial m_{sy}} = 0,\quad \frac{\partial R_{dE,i}}{\partial \omega_{sy}} = 0,\quad \frac{\partial R_{dE,i}}{\partial \xi_{sy}} = 0,\quad \frac{\partial R_{dE,i}}{\partial d_{sy}} = 0 \tag{7}$$

to minimize $R_{dE,i}$. Lin *et al.* (1994, 1995, 2001) found that the optimal PTMD mass, $(m_{sy})_{opt}$, exists, but is rarely used because of economic considerations. Hence, $(\omega_{sy}, \xi_{sy}, d_{sy})_{opt}$ is usually calculated for a known m_{sy} in general cases.

As is evident in the theoretical derivation, a PTMD is optimally designed to control the modes that make the greatest contribution to a specified DOF of the primary structure. For a torsionally coupled building, the first three modes generally dominate the translational and torsional responses of each floor. Therefore, it is possible to reduce the dynamic responses of all DOFs using one PTMD. However, because a single PTMD cannot provide the optimal control for each DOF, the DOF with the largest dynamic response, generally the weakest direction among the x, y and θ axes of the top floor, is minimized when designing a single PTMD.

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Table 1. Physical parameters of a one-story building

Building	Floor mass m (kg)	Story stiffness			Radius of gyration (m)
		k_x (N/m)	k_y (N/m)	k_θ (Nm)	
Square building	$2{\cdot}8 \times 10^5$	$3{\cdot}40 \times 10^7$	$3{\cdot}20 \times 10^7$	$3{\cdot}60 \times 10^9$	8·0

Table 2. Modal properties for one-story torsionally coupled buildings

Building	Mode shape			Frequency (Hz)	Damping ratio (%)
	Mode 1	Mode 2	Mode 3		
$e/r = 0{\cdot}0$	0·000	1·000	0·000	1·701	2·000
	1·000	0·000	0·000	1·704	2·000
	0·000	0·000	1·000	2·256	2·071
$e/r = 0{\cdot}1$	1·000	1·000	1·000	1·688	2·000
	−4·759	0·228	−0·868	1·744	2·000
	0·733	0·115	−6·998	2·287	2·083
$e/r = 0{\cdot}2$	1·000	1·000	1·000	1·641	2·000
	−1·780	0·602	−0·878	1·734	2·000
	0·624	0·115	−4·105	2·367	2·115
$e/r = 0{\cdot}3$	1·000	1·000	1·000	1·571	2·000
	−1·339	0·791	−0·889	1·730	2·000
	0·660	0·090	−3·317	2·477	2·166
$e/r = 0{\cdot}4$	1·000	1·000	1·000	1·494	2·000
	−1·198	0·876	−0·898	1·729	2·000
	0·686	0·071	−3·023	2·607	2·233
$e/r = 0{\cdot}5$	1·000	1·000	1·000	1·417	2·000
	−1·133	0·918	−0·905	1·728	2·000
	0·695	0·058	−2·915	2·749	2·315

4. PARAMETRIC STUDY FOR PTMD DESIGN

There are three important issues in PTMD design: the location of the installation of the PTMD, the detuning effect, and the number of PTMDs. The structural modes dominating the controlled DOF affect the PTMD planar position and the detuning effect. The degree of coupling among the DOFs will determine the number of PTMDs. A one-story torsionally coupled building, called the 'square building', with various normalized eccentricity ratios (e/r) ranging from 0·0 (uncoupled building) to 0·5 (highly coupled building) in both the x- and y-axes, is used to demonstrate this issue. The 'square building' has nearly the same stiffness in the x and y directions ($k_x/k_y \approx 1$) with physical parameters listed in Table 1. Rayleigh damping and 2% damping ratio were assumed for the first and second modes. The first three modal properties for this square building with e/r = 0·0, 0·1, 0·2, 0·3, 0·4 and 0·5 are listed in Table 2.

The mean square displacement response ratios that depict the coupling between DOF x and y are shown in Figure 3. The larger the eccentricity ratio, the higher the coupling between the two translations.

It is well known that vibration control in structures using PTMDs is attributed mainly to the suppression of the controlled modal responses. Because the optimal PTMD installation location and movement direction are related to the mode shape values, the determination of the controlled mode for a

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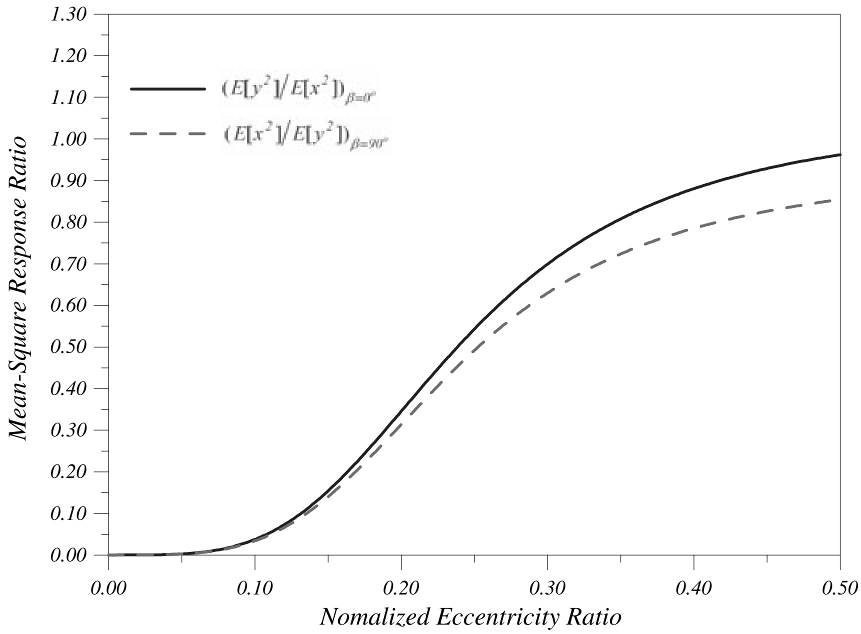

Figure 3. Mean square displacement response ratio of square buildings with different eccentricities

specified DOF is crucial for the PTMD design. By calculating the effective modal mass ratio E_j for each mode j, which is expressed as

$$E_j = \frac{M_j \Gamma_j^2}{M_T} \tag{8}$$

where $M_j = \boldsymbol{\phi}_j^T \boldsymbol{M} \boldsymbol{\phi}_j$, M_T is the total mass of the structure, and Γ_j represents the modal participation coefficient of the jth mode, the dominant mode can be determined. The summation of E_j for all modes is equal to *1*. The effective modal mass ratio for each mode with different e/r values, for this building ($e_x = e_y = e \neq 0$) under x- and y-directional base excitation, is shown in Figure 4. The following findings are made:

(1) The second mode is the controlled mode of the x-response when the building is subjected to x-directional base excitation.
(2) The first mode is the controlled mode of the y-response when the building is subjected to y-directional base excitation.
(3) There is only one controlled mode when e/r is small, but when e/r has a large value two controlled modes occur.

4.1 PTMD installation location

Lin *et al.* (1995) showed that for a symmetrical building the floor corresponding to the tip of the controlled mode shape will be the optimum PTMD location, because it will be able to achieve the greatest response reduction. Similarly, for a torsionally coupled building with a weak y-axis, the PTMD is optimally installed on the top floor, moving in the y direction. The optimum planar position for the PTMD can be determined by maximizing the absolute values of $(\phi_{3l-1,j} + d_{sy}\phi_{3l,j})$ for moving in the y

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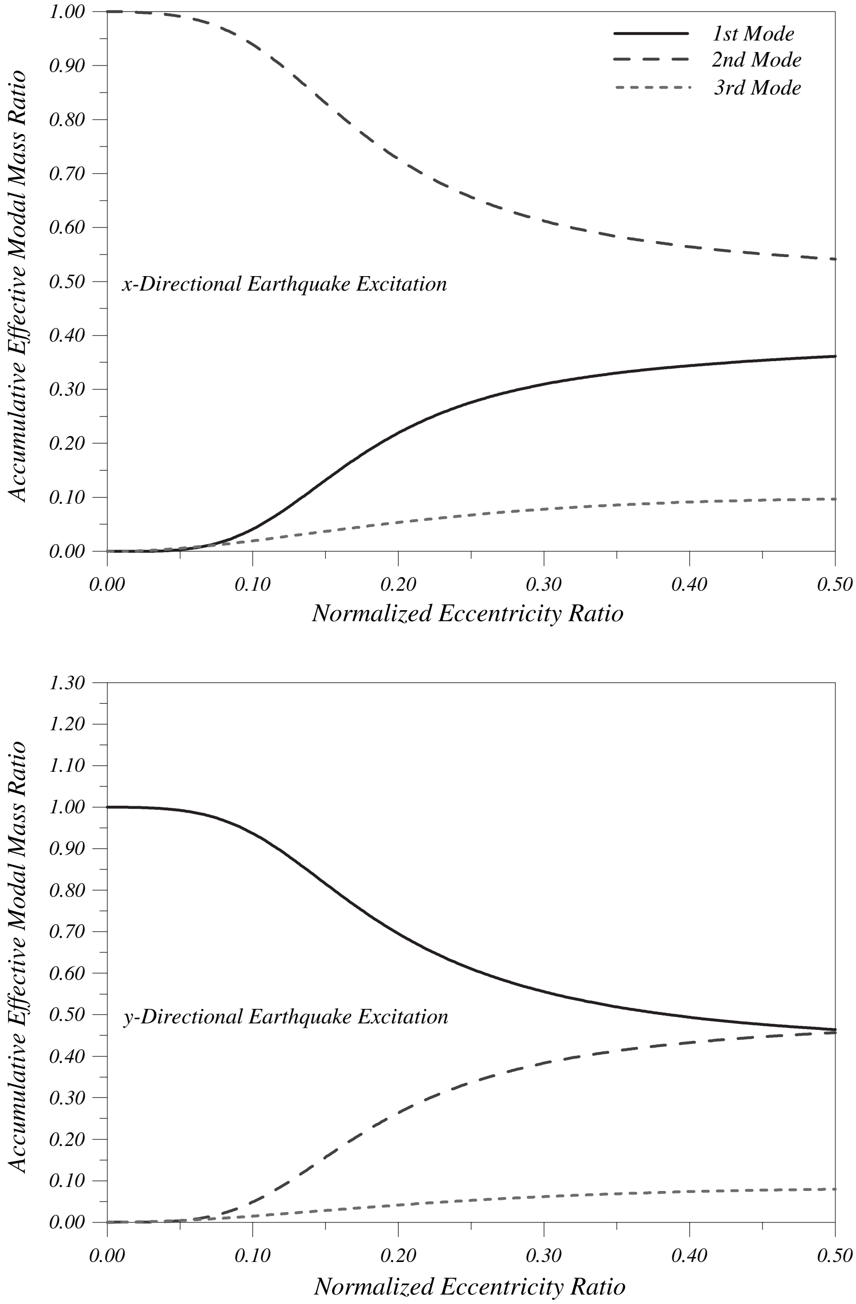

Figure 4. Variation of effective modal mass ratio for square buildings with different eccentricities

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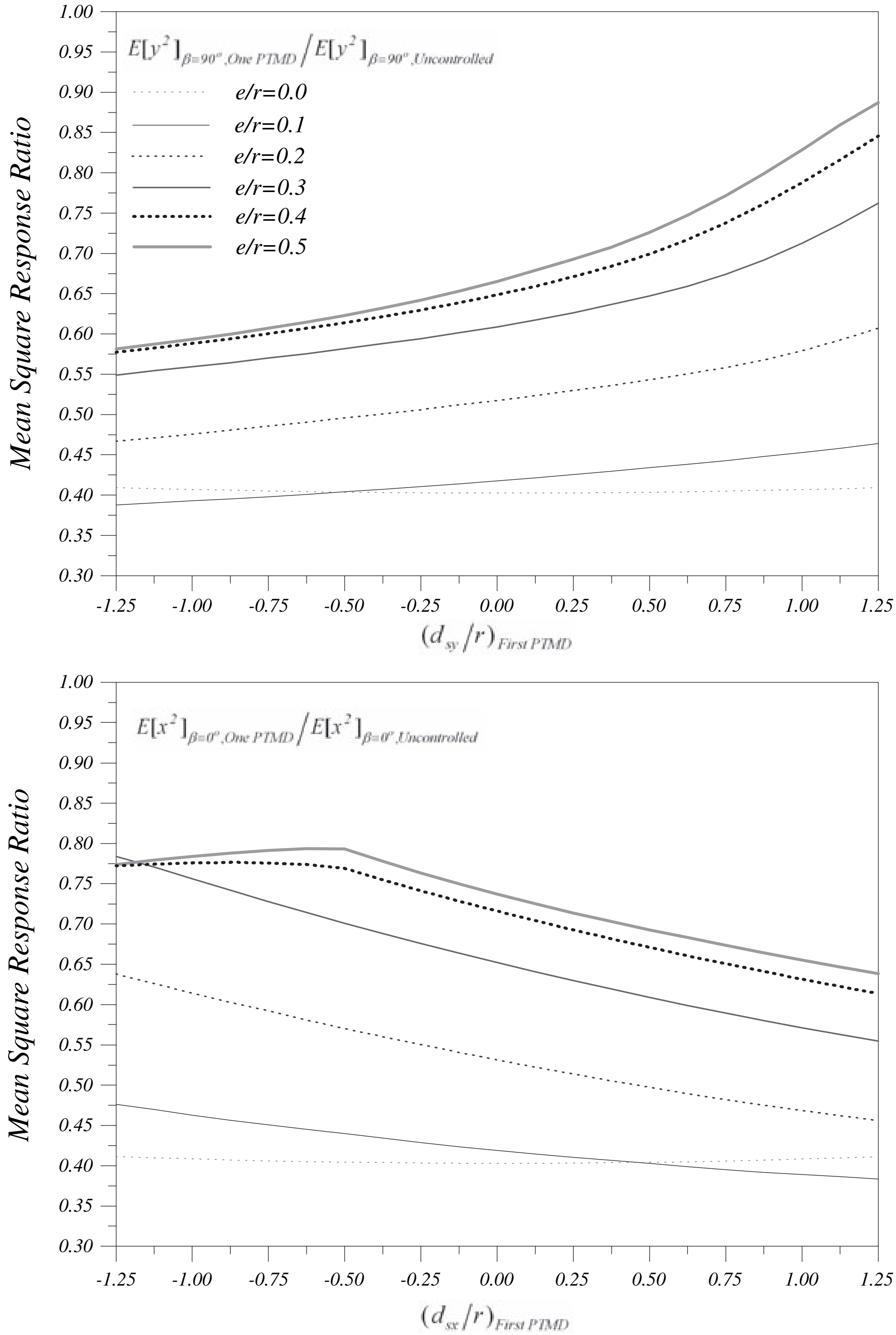

Figure 5. Mean square displacement response ratio with PTMD's planar position

direction, or $(\phi_{3l-2,j} + d_{sx}\phi_{3l,j})$ for moving in the x direction. The absolute value for $(\phi_{3l-1,j} + d_{sy}\phi_{3l,j})$ and $(\phi_{3l-2,j} + d_{sx}\phi_{3l,j})$ depends on the translation sign ($\phi_{3l-1,j}$ or $\phi_{3l-2,j}$) and the rotation ($\phi_{3l,j}$) mode shape values. When both mode shape values have the same signs, the maximum allowable positive value for d_{sy} or d_{sx} is chosen for the installed floor. On the other hand, when $\phi_{3l-1,j}$ or $\phi_{3l-2,j}$ and $\phi_{3l,j}$ have opposite signs, we choose d_{sy} or d_{sx} to be the maximum negative value.

From the above, it can be concluded that the greater the distance between the PTMD and the mass center of the installation floor, the greater the vibration reduction obtained. A one-story building with various eccentricity ratios is used to discuss the effect of the PTMD planar position on the response control efficacy. The total mass ratio of the PTMD to building is set at 2%, and the allowable range of PTMD planar position is $-1{\cdot}25 \leq d_{sy}/r \leq 1{\cdot}25$.

There are two design cases used to discuss the optimum PTMD planar position. In the first case, the PTMD is designed to move in the y direction to minimize its mean square response $[E(y^2)]$ when the building is subjected to y-directional earthquake excitation ($\beta = 90°$). In the second case, the PTMD moves in the x direction to minimize its mean square response $[E(x^2)]$ for x-directional earthquake excitation ($\beta = 0°$). The mean square displacement response ratio versus PTMD planar position for $e/r = 0{\cdot}0$, $0{\cdot}1$, $0{\cdot}2$, $0{\cdot}3$, $0{\cdot}4$ and $0{\cdot}5$ are shown in Figure 5. The following observations are made:

- *Case 1*: When a building is subjected to y-directional earthquake excitation, the first mode makes the greatest contribution to the y response. Because $\phi_{2,1}$ and $\phi_{3,1}$ of the first mode have opposite signs for this kind of building, as listed in Table 2, the PTMD planar position d_{sy} can be determined by maximizing the absolute values for $(\phi_{2,1} + d_{sy}\phi_{3,1})$ in Equation (4), to achieve the most effective response control. For the cases of $e/r > 0$, d_{sy} is chosen as the maximum negative value corresponding to the maximum allowable distance on the opposite side of the resistance center, and there is a minimum mean square displacement response ratio. When $e/r = 0$, the mean square displacement response ratio is the same for a given PTMD planar position d_{sy}. This is because the building is uncoupled in three DOFs, i.e. $\phi_{3,1} = 0$.
- *Case 2*: Similarly, when a building is subjected to x-directional earthquake excitation, the second mode makes the greatest contribution to the x response. Because $\phi_{1,2}$ and $\phi_{3,2}$ of the second mode have the same signs for this kind of building, as listed in Table 2, the PTMD planar position d_{sx} can be determined by maximizing the absolute values for $(\phi_{1,2} + d_{sx}\phi_{3,2})$ to achieve the most effective response control. For the cases of $e/r > 0$, d_{sx} is chosen as the maximum positive value, which corresponds to the maximum allowable distance on the opposite side of the resistance center. When $e/r = 0$ (uncoupled in three DOFs), the mean square displacement response ratios are the same for various PTMD planar positions d_{sx}. This implies that the PTMD control effectiveness is not affected by the PTMD planar position.

From the above numerical verifications, the greater the distance between the PTMD and the mass center of the installed floor, the greater the PTMD control effectiveness. For conventional PTMD design, the torsionally coupled effect of a building was generally neglected, and the PTMD was installed at the center of mass. To compare the control performance for a PTMD installed at the optimal planar position with that installed at the center of mass, the mean square displacement responses versus e/r for a 2% PTMD mass ratio moving in the y direction are shown in Figure 6. It is evident that the PTMD in the optimal planar position has better response control efficacy than the PTMD at the center of mass, especially for larger e/r values.

4.2 PTMD detuning effect

If the PTMD parameters shift away from their optimum values, the response control effectiveness is expected to degrade. Using the previous one-story torsionally coupled building, the mean square

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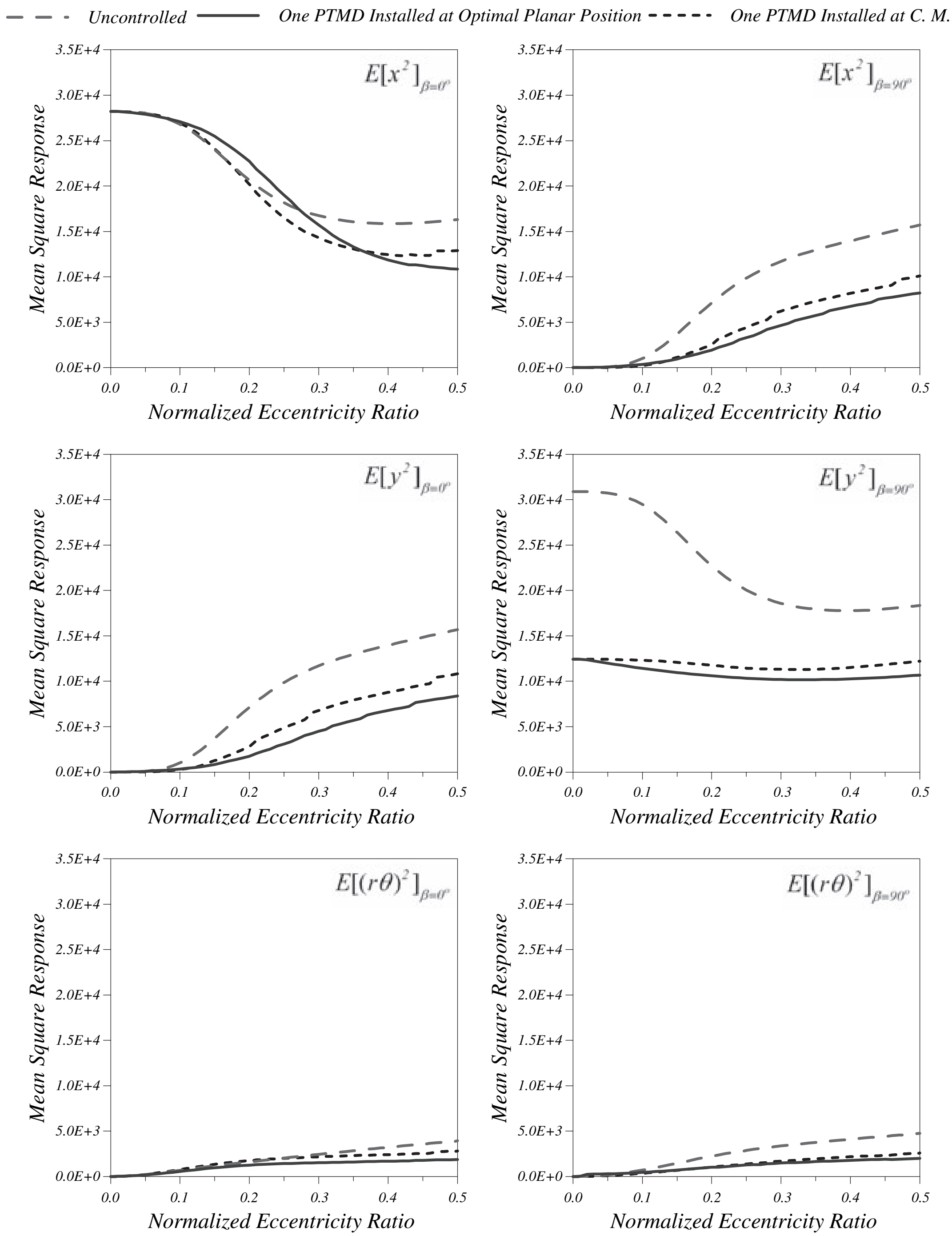

Figure 6. Mean square displacement response ratio with and without PTMD

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displacement response ratio under a given variation of the PTMD frequency and damping ratio was calculated to show the PTMD detuning effect. Assuming that a PTMD with 2% mass ratio is placed at the optimal planar position and moves in the *y* direction, the mean square displacement response ratio, with the PTMD frequency and damping ratio shifted from their optimum values by ±10% and ±20% respectively, and are shown in Figure 7 for various *e*/*r* values. It was found that the PTMD frequency detuning effect on the response reduction is more significant than the PTMD damping ratio. Moreover, the maximum detuning error, which is defined as the relative error percentage of the maximum mean square response ratio to the minimum mean square response in the detuning range considered, decreases with an increasing *e*/*r*. Hence, for square buildings with low-coupled DOFs, a significant variation in response control occurs when the PTMD parameters shift away from their respective optimum values. Conversely, less effect is found for highly coupled square buildings.

4.3 PTMD numbers

According to the above design procedure for one PTMD, the dynamic response of controlled DOF response is reduced. Because this PTMD is designed based on the dominant modal properties of this DOF, its capability in reducing the responses of other DOFs under an earthquake from different angles should be further investigated. This study found that whether additional PTMDs are required depends upon the degree of coupling among the DOFs. For instance, a square building with nearly equal stiffness and small static eccentricities in the *x* and *y* directions has slight coupling in the *x* and *y* responses although both translational frequencies are close. One PTMD designed for reducing the *y* responses is not able to decrease the *x* responses if an earthquake is applied from the *x*-response critical incident angle and vice versa. Under this circumstance, a second PTMD installed in the *x* direction to control the *x* responses becomes necessary.

From Figure 6, which depicts the mean square displacement responses with different *e*/*r* values for the above one-story torsionally coupled square building with and without PTMD, it is evident that when the building is subjected to *y*-directional earthquake excitation the *y*-directional mean square displacement response is suppressed significantly for different *e*/*r* values by a PTMD installed in the *y* direction. As the building is subjected to *x*-directional earthquake excitation, the *x*-directional mean square displacement response is amplified for small *e*/*r* values by the *y*-directional PTMD. This is attributed to the *x*-displacement dominant modal response amplification (2nd mode) after the first PTMD is installed. In this case, a second PTMD installed in the *x* direction to control the *x* responses is required. However, for the square building with large *e*/*r*, one *y*-directional PTMD is effective in reducing the *x*-directional responses. This is because a building with larger *e*/*r* values possesses highly coupled translational and rotational responses. This new finding is different from the general practice that asks for two PTMDs to be installed in both the *x* and *y* directions.

5. OPTIMAL DESIGN OF A SECOND PTMD

From the discussion in the previous section, a second PTMD is required to suppress the vibration response for some buildings when they are subjected to earthquake excitation from other directions. Assume that a second PTMD with mass m_{sx}, damping c_{sx}, and stiffness k_{sx}, is also mounted on the *l*th floor of the *N*-story torsionally coupled building to control the *x*-directional response and moving in the *x* direction, u_{sx} and u_{sy} denote the two PTMD displacements relative to the base, and d_{sx} and d_{sy} are the distances between the two PTMDs to the *x*-axis and *y*-axis, respectively, as shown in Figure 2. $v_{sx} = u_{sx} - (x_l + d_{sx}\theta_l)$ and $v_{sy} = u_{sy} - (y_l + d_{sy}\theta_l)$ are the displacements of the two PTMDs relative to the floor, and $\omega_{sx} = \sqrt{k_{sx}/m_{sx}}$, $\omega_{sy} = \sqrt{k_{sy}/m_{sy}}$ and $\xi_{sx} = c_{sx}/(2m_{sx}\omega_{sx})$, $\xi_{sy} = c_{sy}/(2m_{sy}\omega_{sy})$ represent the natural

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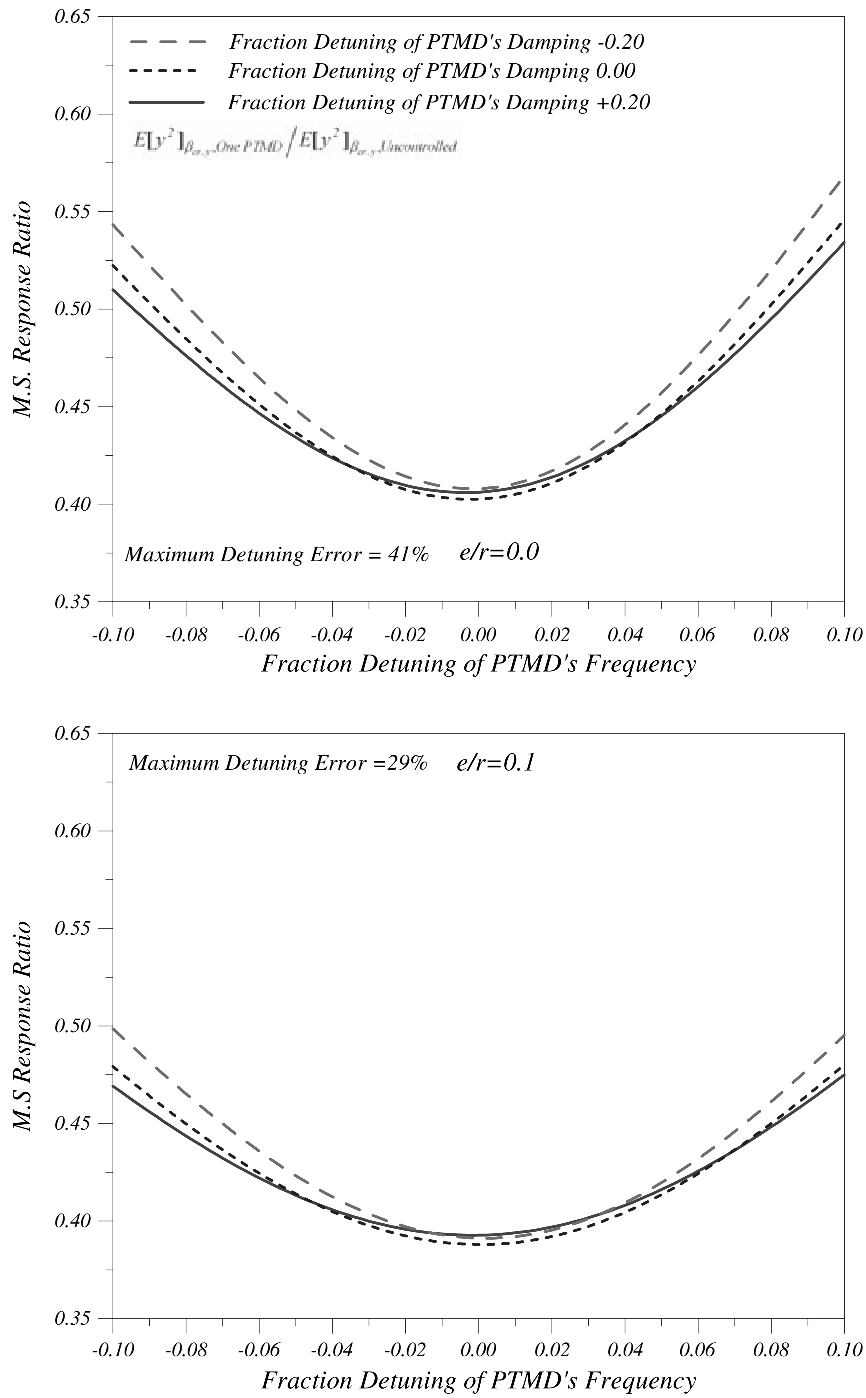

Figure 7. Effect of detuning: mean square displacement response ratio

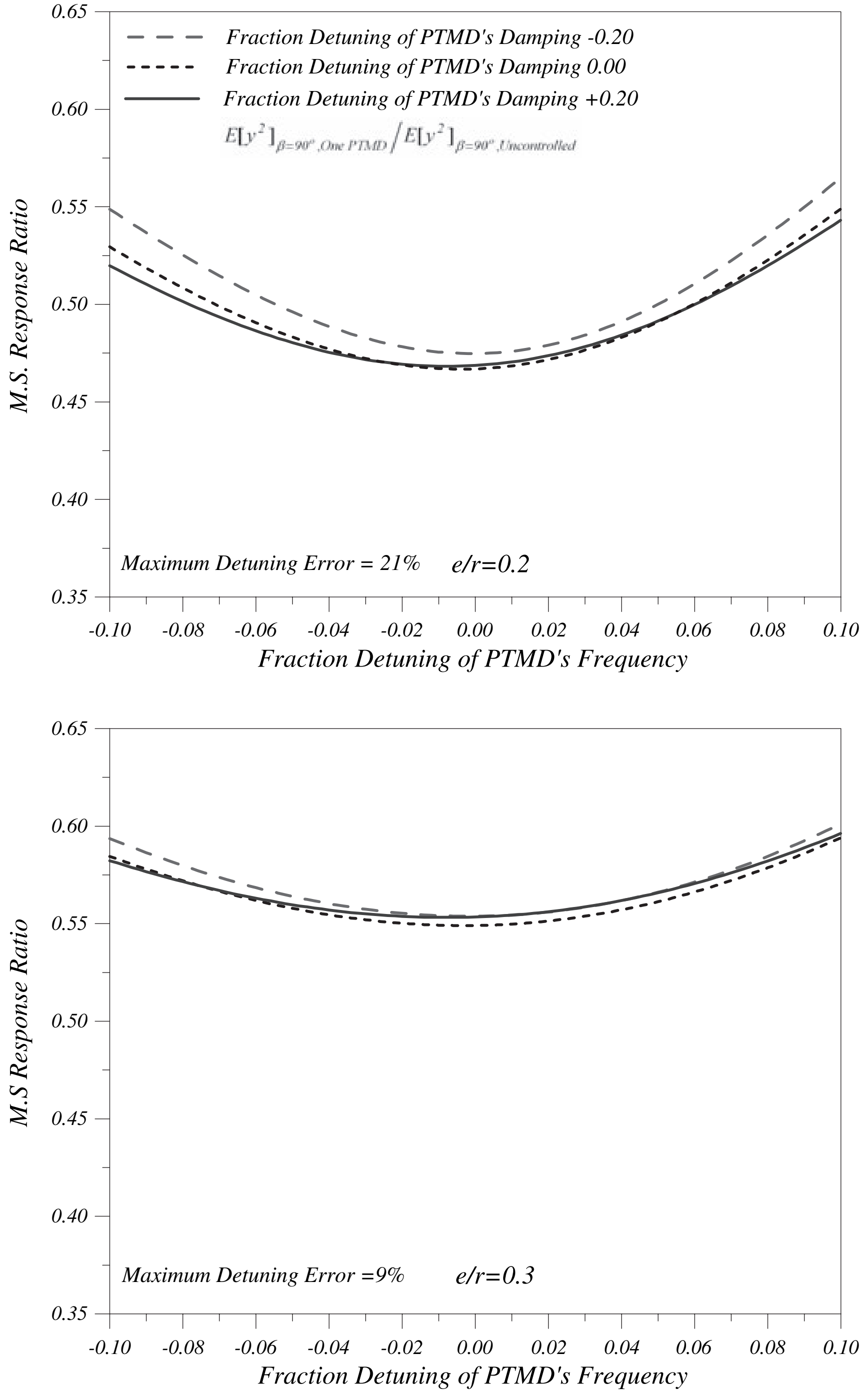

Figure 7. *Continued*

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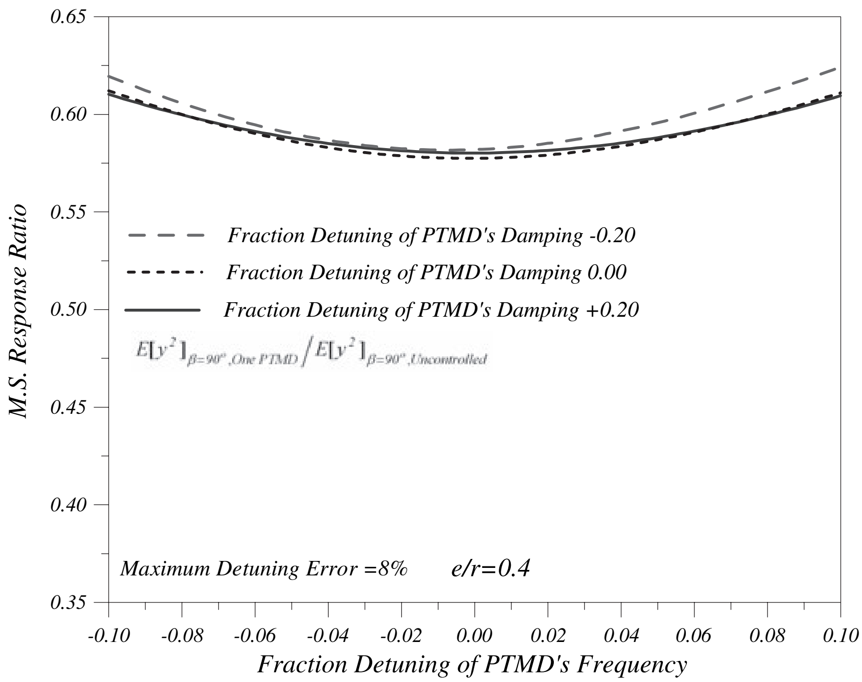

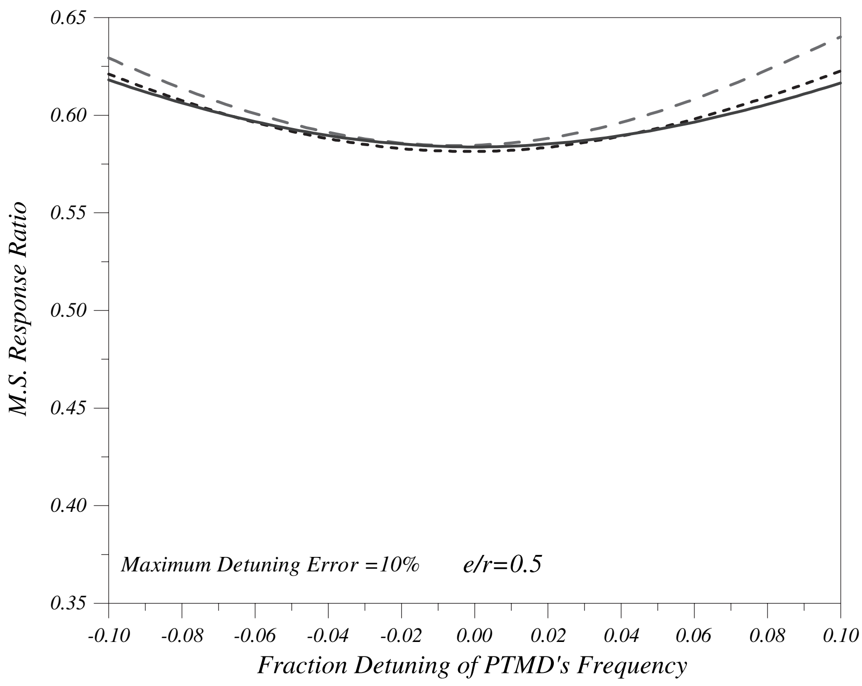

Figure 7. *Continued*

lth frequency and damping ratio of the two PTMDs, respectively. The equations of motion for the first three modes of primary structure and PTMDs are expressed in matrix form as

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\phi_{3l-2,1}+d_{sx}\phi_{3l,1} & \phi_{3l-2,2}+d_{sx}\phi_{3l,2} & \phi_{3l-2,3}+d_{sx}\phi_{3l,3} & 1 & 0 \\
\phi_{3l-1,1}+d_{sy}\phi_{3l,1} & \phi_{3l-1,2}+d_{sy}\phi_{3l,2} & \phi_{3l-1,3}+d_{sy}\phi_{3l,3} & 0 & 1
\end{bmatrix}
\begin{Bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \\ \ddot{\eta}_3 \\ \ddot{v}_{sx} \\ \ddot{v}_{sy} \end{Bmatrix}
+
\begin{bmatrix}
2\xi_1\omega_1 & 0 & 0 & -2\mu_{1x}\xi_{sx}\omega_{sx} & -2\mu_{1y}\xi_{sy}\omega_{sy} \\
0 & 2\xi_2\omega_2 & 0 & -2\mu_{2x}\xi_{sx}\omega_{sx} & -2\mu_{2y}\xi_{sy}\omega_{sy} \\
0 & 0 & 2\xi_3\omega_3 & -2\mu_{3x}\xi_{sx}\omega_{sx} & -2\mu_{3y}\xi_{sy}\omega_{sy} \\
0 & 0 & 0 & 2\xi_{sx}\omega_{sx} & 0 \\
0 & 0 & 0 & 0 & 2\xi_{sy}\omega_{sy}
\end{bmatrix}
\begin{Bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \\ \dot{v}_{sx} \\ \dot{v}_{sy} \end{Bmatrix}
$$

$$
+
\begin{bmatrix}
\omega_1^2 & 0 & 0 & -\mu_{1x}\omega_{sx}^2 & -\mu_{1y}\omega_{sy}^2 \\
0 & \omega_2^2 & 0 & -\mu_{2x}\omega_{sx}^2 & -\mu_{2y}\omega_{sy}^2 \\
0 & 0 & \omega_3^2 & -\mu_{3x}\omega_{sx}^2 & -\mu_{3y}\omega_{sy}^2 \\
0 & 0 & 0 & \omega_{sx}^2 & 0 \\
0 & 0 & 0 & 0 & \omega_{sy}^2
\end{bmatrix}
\begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ v_{sx} \\ v_{sy} \end{Bmatrix}
=
\begin{Bmatrix}
-\dfrac{1}{m_1^*}(\phi_1^T \boldsymbol{M}_p \boldsymbol{r}) \\
-\dfrac{1}{m_2^*}(\phi_2^T \boldsymbol{M}_p \boldsymbol{r}) \\
-\dfrac{1}{m_3^*}(\phi_3^T \boldsymbol{M}_p \boldsymbol{r}) \\
-\cos\beta \\
-\sin\beta
\end{Bmatrix} \ddot{u}_g \tag{9}
$$

Following the same optimization procedure as in the preceding section, the optimal system parameters for the second PTMD are determined by minimizing the mean square displacement response ratio of the second controlled DOF k which has the largest x responses, $R_{dE,k}$, between the building with two PTMDs and the building with one PTMD under an earthquake from the x direction.

The mean square displacement responses for the above square building with $e/r = 0$ to 0·5 and two PTMDs with 1% mass ratio in both the y and x directions are shown in Figure 8. It was found that when the building is subjected to an x-directional earthquake the x-directional response is significantly suppressed using two PTMDs. As to the building subjected to a y-directional earthquake, one PTMD with a 2% mass ratio has better control than two PTMDs with 1% mass ratio. The response control performance of two PTMDs installed at the optimal planar position is also compared with those at the center of mass, as shown in Figure 8. As expected, the PTMDs in the optimal locations produced better response control efficacy than those at the center of mass.

6. NUMERICAL VERIFICATIONS OF OPTIMAL PASSIVE TUNED MASS DAMPERS

Two five-story torsionally coupled buildings, a square building with small eccentricity (B1) and a square building with large eccentricity (B2), are used to demonstrate the new design procedure and vibration control effectiveness of the proposed optimal PTMDs. These buildings (B1, B2) have nearly the same stiffness in the x and y directions as a square building. B1 has small static eccentricities in both the x- and y-axes that cause low-coupled responses in the three DOFs of each floor. B2 has large static eccentricities in both the x- and y-axes that cause highly coupled responses in the three DOFs

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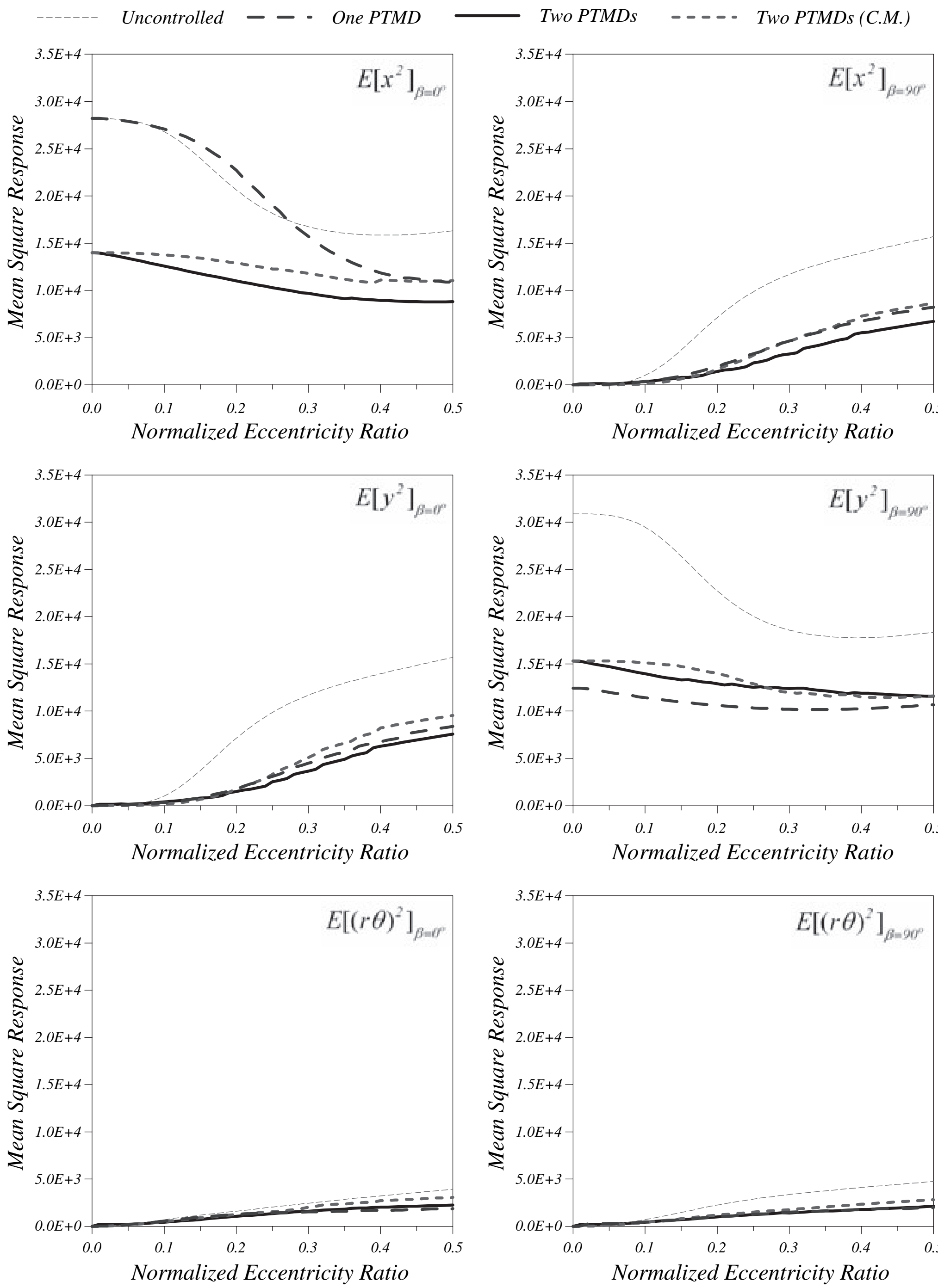

Figure 8. Mean square displacement response with and without PTMDs

Table 3. Physical system parameters of building B1

Building		Floor mass m (kg)	Story stiffness k_x (N/m)	k_y (N/m)	k_θ (Nm)	Story eccentricity (m) e_x	e_y	Radius of gyration (m)
B1	1F	$2{\cdot}8 \times 10^5$	$3{\cdot}21 \times 10^8$	$3{\cdot}20 \times 10^8$	$3{\cdot}60 \times 10^{10}$	0·8	0·8	8·0
	2F	$2{\cdot}6 \times 10^5$	$3{\cdot}16 \times 10^8$	$3{\cdot}15 \times 10^8$	$3{\cdot}55 \times 10^{10}$	0·8	0·8	8·0
	3F	$2{\cdot}4 \times 10^5$	$3{\cdot}11 \times 10^8$	$3{\cdot}10 \times 10^8$	$3{\cdot}50 \times 10^{10}$	0·8	0·8	8·0
	4F	$2{\cdot}2 \times 10^5$	$3{\cdot}06 \times 10^8$	$3{\cdot}05 \times 10^8$	$3{\cdot}45 \times 10^{10}$	0·8	0·8	8·0
	5F	$2{\cdot}0 \times 10^5$	$3{\cdot}01 \times 10^8$	$3{\cdot}00 \times 10^8$	$3{\cdot}40 \times 10^{10}$	0·8	0·8	8·0

Table 4. Modal properties of building B1

	Mode shape x component	y component	$r\theta$ component	Frequency (Hz)	Damping ratio (%)
Mode 1	$\begin{Bmatrix} 0{\cdot}291 \\ 0{\cdot}556 \\ 0{\cdot}773 \\ 0{\cdot}924 \\ 1{\cdot}000 \end{Bmatrix}$	$\begin{Bmatrix} -0{\cdot}329 \\ -0{\cdot}630 \\ -0{\cdot}875 \\ -1{\cdot}046 \\ -1{\cdot}132 \end{Bmatrix}$	$\begin{Bmatrix} 0{\cdot}077 \\ 0{\cdot}148 \\ 0{\cdot}205 \\ 0{\cdot}245 \\ 0{\cdot}265 \end{Bmatrix}$	1·680	2·000
Mode 2	$\begin{Bmatrix} 0{\cdot}291 \\ 0{\cdot}556 \\ 0{\cdot}773 \\ 0{\cdot}924 \\ 1{\cdot}000 \end{Bmatrix}$	$\begin{Bmatrix} 0{\cdot}258 \\ 0{\cdot}493 \\ 0{\cdot}686 \\ 0{\cdot}819 \\ 0{\cdot}887 \end{Bmatrix}$	$\begin{Bmatrix} 0{\cdot}004 \\ 0{\cdot}008 \\ 0{\cdot}012 \\ 0{\cdot}014 \\ 0{\cdot}015 \end{Bmatrix}$	1·702	2·000
Mode 3	$\begin{Bmatrix} 0{\cdot}290 \\ 0{\cdot}555 \\ 0{\cdot}772 \\ 0{\cdot}924 \\ 1{\cdot}000 \end{Bmatrix}$	$\begin{Bmatrix} -0{\cdot}288 \\ -0{\cdot}551 \\ -0{\cdot}767 \\ -0{\cdot}917 \\ -0{\cdot}993 \end{Bmatrix}$	$\begin{Bmatrix} -2{\cdot}329 \\ -4{\cdot}455 \\ -6{\cdot}186 \\ -7{\cdot}388 \\ -7{\cdot}993 \end{Bmatrix}$	2·285	2·091

of each floor. The physical and the first three modal parameters of both buildings are listed in Tables 3–6. The total mass ratio for one PTMD or two PTMDs to the building's total mass is set at 2% and $-10\,m \le d_{sx}, d_{sy} \le 10\,m$ in the following numerical examples. The normalized (PGA = $0{\cdot}3\,g$) 1979 El Centro earthquakes (S50W and S40E) were used as the bidirectional earthquake inputs to verify the vibration control efficacy.

6.1 Square building with small eccentricity (B1)

For this building, the y direction is less stiff and is thus selected as the direction of desired controlled DOF and PTMD movement direction. Hence, the PTMD is installed in the y direction on the top floor to control the y response on the same floor. The optimal PTMD's system parameters and locations were calculated using Equation (7) and are listed in Table 7. Figure 9 depicts the transfer functions for the top floor displacement in three directions for earthquake excitation from $\beta = 0°$ and 90°. Figure 9 shows that the PTMD is tuned mainly to the first mode. Because $\phi_{14,1}$ and $\phi_{15,1}$ of the first mode have different signs (i.e. $-1{\cdot}132$ and $0{\cdot}262$), d_{sy} is optimally designed at $-10\,m$, which corresponds to the

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Table 5. Physical system parameters for building B2

Building		Floor mass m (kg)	Story stiffness			Story eccentricity (m)		Radius of gyration (m)
			k_x (N/m)	k_y (N/m)	k_θ (N–m)	e_x	e_y	
B2	1F	$2{\cdot}8 \times 10^5$	$4{\cdot}00 \times 10^8$	$3{\cdot}99 \times 10^8$	$3{\cdot}00 \times 10^{10}$	2·4	2·4	8·0
	2F	$2{\cdot}6 \times 10^5$	$3{\cdot}92 \times 10^8$	$3{\cdot}90 \times 10^8$	$2{\cdot}90 \times 10^{10}$	2·4	2·4	8·0
	3F	$2{\cdot}4 \times 10^5$	$3{\cdot}90 \times 10^8$	$3{\cdot}85 \times 10^8$	$2{\cdot}80 \times 10^{10}$	2·4	2·4	8·0
	4F	$2{\cdot}2 \times 10^5$	$3{\cdot}85 \times 10^8$	$3{\cdot}83 \times 10^8$	$2{\cdot}70 \times 10^{10}$	2·4	2·4	8·0
	5F	$2{\cdot}0 \times 10^5$	$3{\cdot}84 \times 10^8$	$3{\cdot}82 \times 10^8$	$2{\cdot}60 \times 10^{10}$	2·4	2·4	8·0

Table 6. Modal properties for building B2

	Mode shape			Frequency (Hz)	Damping ratio (%)
	x component	y component	$r\theta$ component		
Mode 1	$\begin{Bmatrix} 0{\cdot}289 \\ 0{\cdot}555 \\ 0{\cdot}772 \\ 0{\cdot}924 \\ 1{\cdot}000 \end{Bmatrix}$	$\begin{Bmatrix} -0{\cdot}291 \\ -0{\cdot}561 \\ -0{\cdot}782 \\ -0{\cdot}936 \\ -1{\cdot}013 \end{Bmatrix}$	$\begin{Bmatrix} 0{\cdot}277 \\ 0{\cdot}534 \\ 0{\cdot}749 \\ 0{\cdot}902 \\ 0{\cdot}981 \end{Bmatrix}$	1·600	2·000
Mode 2	$\begin{Bmatrix} 0{\cdot}292 \\ 0{\cdot}560 \\ 0{\cdot}776 \\ 0{\cdot}926 \\ 1{\cdot}000 \end{Bmatrix}$	$\begin{Bmatrix} 0{\cdot}289 \\ 0{\cdot}556 \\ 0{\cdot}773 \\ 0{\cdot}922 \\ 0{\cdot}996 \end{Bmatrix}$	$\begin{Bmatrix} 0{\cdot}002 \\ 0{\cdot}005 \\ 0{\cdot}007 \\ 0{\cdot}008 \\ 0{\cdot}009 \end{Bmatrix}$	1·899	2·000
Mode 3	$\begin{Bmatrix} 0{\cdot}301 \\ 0{\cdot}574 \\ 0{\cdot}787 \\ 0{\cdot}931 \\ 1{\cdot}000 \end{Bmatrix}$	$\begin{Bmatrix} -0{\cdot}295 \\ -0{\cdot}564 \\ -0{\cdot}777 \\ -0{\cdot}919 \\ -0{\cdot}988 \end{Bmatrix}$	$\begin{Bmatrix} -0{\cdot}587 \\ -1{\cdot}036 \\ -1{\cdot}596 \\ -1{\cdot}925 \\ -2{\cdot}095 \end{Bmatrix}$	2·418	2·100

Table 7. Optimal system parameters of one-PTMD system for building B1

Mass ratio (%)	ξ_s (%)	ω_s (Hz)	Installed floor	Moving direction	d_{sy} (m)
2	13·0	1·638	5F	y	−10

maximum allowable distance on the opposite side of the resistance center. The variation in mean square displacement response of the top floor with an earthquake incident angle β ranging from 0° to 180° with and without a PTMD is shown in Figure 10. It is evident that when using one optimal PTMD moving in the y direction the mean square x_5 displacement increases as $\beta = (0–80)°$ and $(170–180)°$. Figure 9 shows that when using one optimal PTMD moving in the y direction the first modal amplitude in all three directional responses is suppressed significantly in the case of $\beta = 90°$. However, in the case of $\beta = 0°$, the second modal amplitude in x_5 displacement is amplified. This is attributed to the x_5 displacement dominant modal response amplification (mode 2) after the

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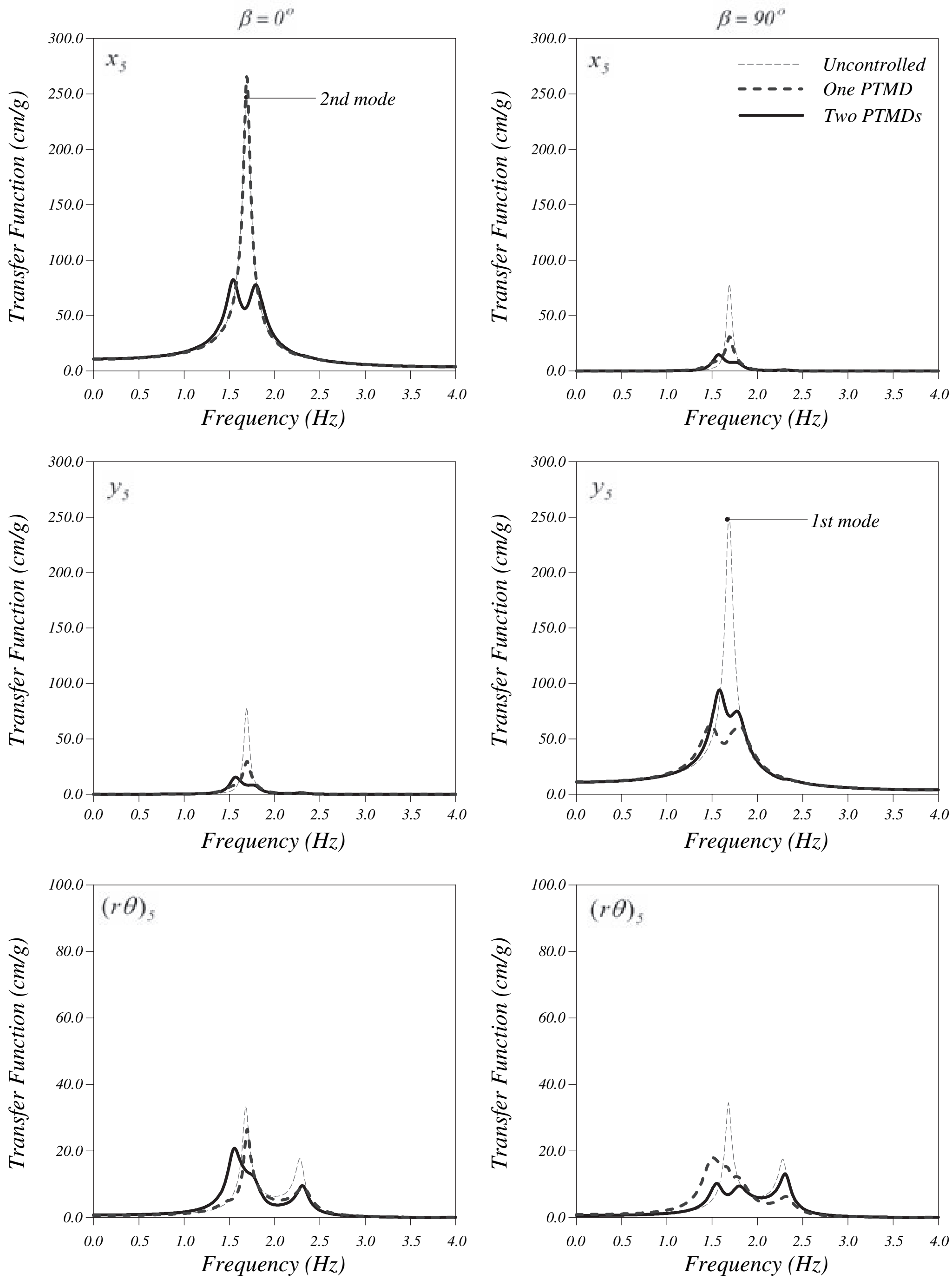

Figure 9. Top floor displacement transfer function for B1

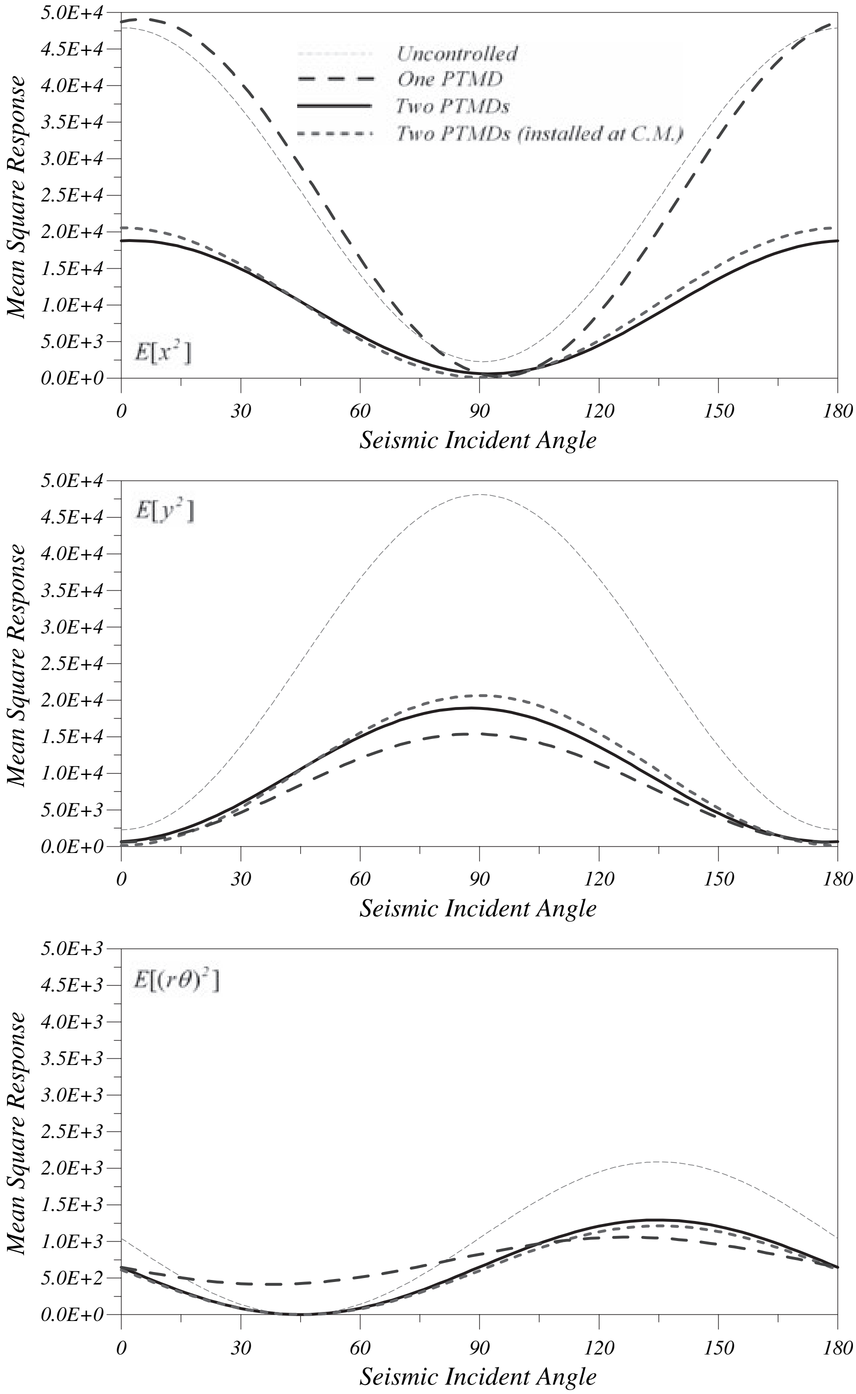

Figure 10. Top floor mean square displacement response of B1 with and without PTMD

Table 8. Optimal system parameters of two-PTMD system of building B1

PTMD	Mass ratio (%)	ξ_s (%)	ω_s (Hz)	Installed floor	Moving direction	d_{sy} or d_{sx} (m)
First	1	8·0	1·674	5F	y	−10 (d_{sy})
Second	1	8·0	1·674	5F	x	10 (d_{sx})

Table 9. Reduction of peak and root-mean-square responses of building B1 under bidirectional 1979 El Centro earthquakes

	Peak response			RMS response		
	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)
Uncontrolled	9·603	9·156	2·975	1·707	1·916	0·444
One PTMD installed at optimal location	8·981	5·382	2·085	1·578	0·954	0·367
	(−6%)	(−41%)	(−30%)	(−8%)	(−50%)	(−17%)
Two PTMDs installed at optimal location	7·330	6·211	2·316	1·275	1·001	0·343
	(−24%)	(−32%)	(−22%)	(−25%)	(−48%)	(−23%)

first PTMD is installed. Thus, a second PTMD installed in the x direction to control the x responses is required.

To compare the vibration control effectiveness between using one and two PTMDs, the same total PTMD mass is used for both cases. The second PTMD was installed in the x direction on the top floor and designed to control x-direction responses. The optimal locations and dynamic parameters for the two-PTMDs system are listed in Table 8. Figure 9 shows that the second PTMD is tuned to the second mode. Because $\phi_{13,2}$ and $\phi_{15,2}$ of the second mode have the same signs (i.e. 1·000 and 0·015), d_{sx} is optimally designed at 10 m, which corresponds to the maximum allowable distance on the opposite side of the resistance center. Figure 9 shows that in both cases of $\beta = 0°$ and $\beta = 90°$ the first two modal amplitudes in all three directional responses are suppressed significantly in the case of using two PTMDs. Simultaneously, the mean square displacement responses of the top floor with two PTMDs, shown in Figure 10, are also reduced in three directions from different earthquake incident angles. The tremendous reduction in the three directional responses again reveals the necessity and importance of the second PTMD.

A seismic effectiveness study was performed for building B1 under the 1979 El Centro bidirectional earthquake ground accelerations. The time history displacement responses at the top floor with one PTMD, with two PTMDs and without PTMD are shown in Figure 11. The peak and root-mean-square top floor displacements in three directions are also summarized in Table 9. Note that the numbers in parentheses denote the percentage of response reduction. As shown in Table 9, the x response is almost not suppressed (reduced only to 6%) using one PTMD. However, both peak and root-mean-square responses are reduced up to 22% in the two-PTMD case. This result indicates that a two-PTMD system produces a much better response control performance for reducing both floor translation and rotation than a one-PTMD system. A second PTMD is therefore required for a square building with small eccentricity.

A peak displacement response ratio analysis, shown in Figure 12, for building B1 installed with two PTMDs and under the bidirectional 1979 El Centro earthquake, was performed to study the detuning effect for the PTMD frequency and damping ratio. For this type of square building with small eccentricity, the response control efficacy has obvious variations when the PTMD parameters shift away from their optimum values.

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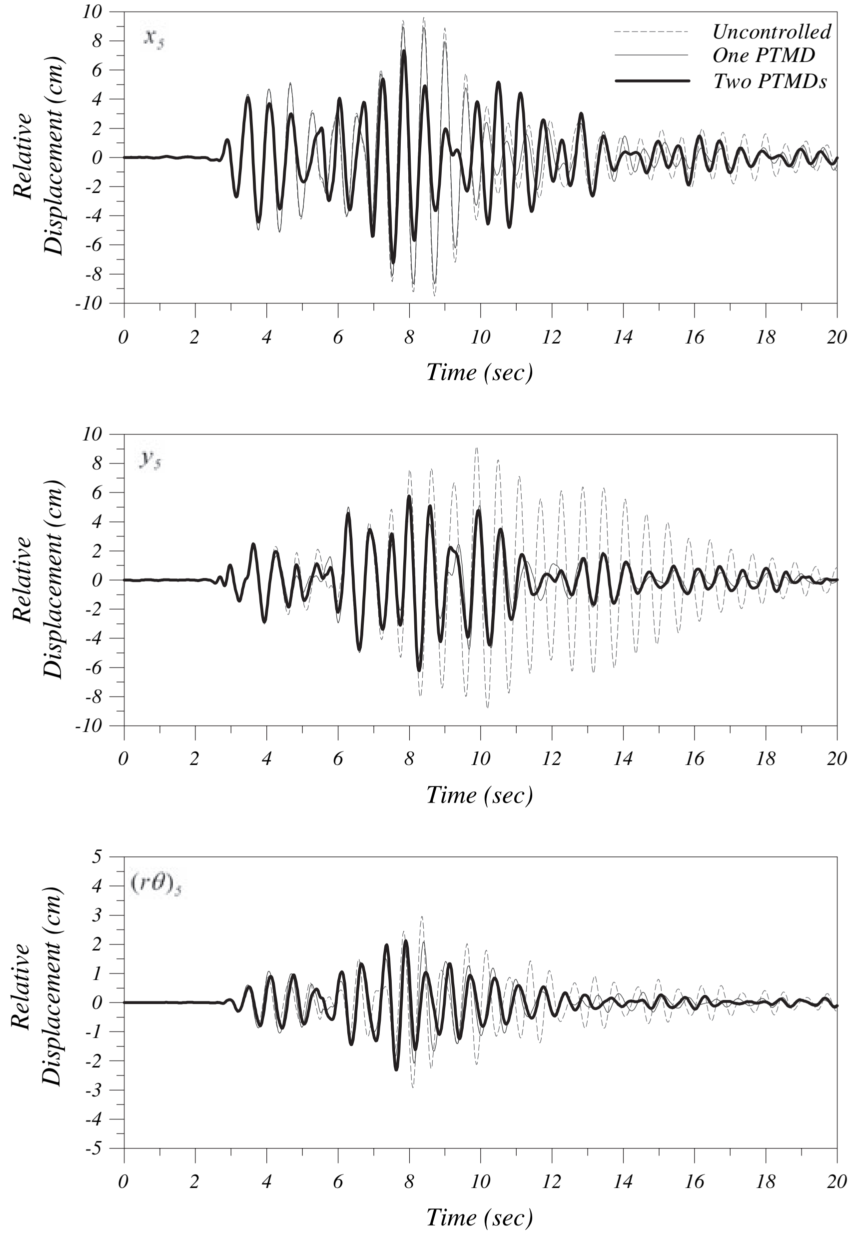

Figure 11. Top floor displacement response of B1 under bidirectional 1979 El Centro earthquakes

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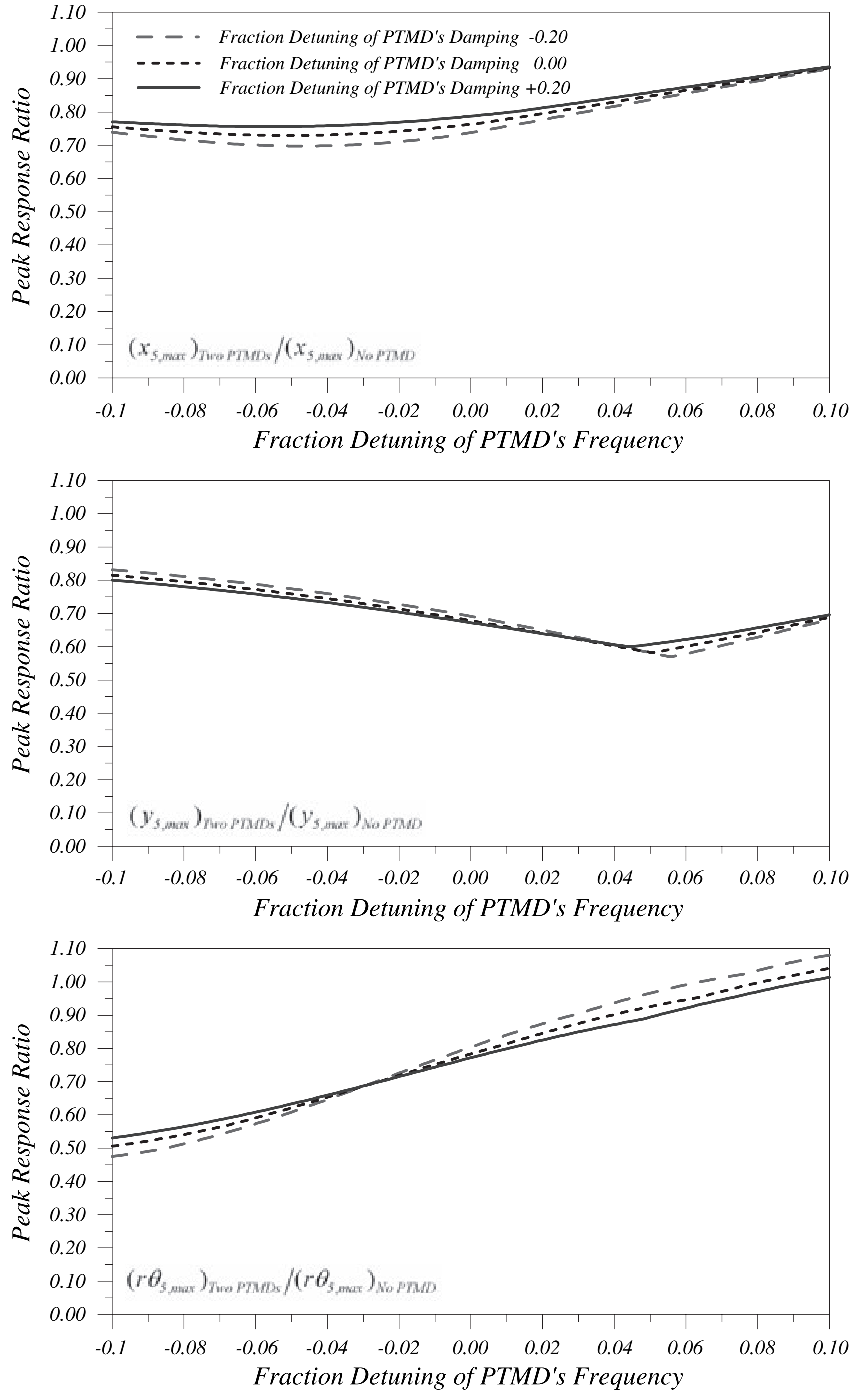

Figure 12. Detuning effect on the displacement response ratio of the top floor of B1

Table 10. Optimal system parameters of the one-PTMD system for building B2

Mass ratio (%)	ξ_s (%)	ω_s (Hz)	Installed floor	Moving direction	d_{sy} (m)
2	20·0	1·660	5F	y	−10

Table 11. Optimal system parameters of the two-PTMDs system for building B2

PTMD	Mass ratio (%)	ξ_s (%)	ωb_s (Hz)	Installed floor	Moving direction	d_{sy} or d_{sx} (m)
First	1	17·0	1·660	5F	y	−10 (d_{sy})
Second	1	6·0	2·002	5F	x	10 (d_{sx})

Table 12. Reduction of peak and root-mean-square responses of building B2 under 1979 El Centro bidirectional earthquakes

	Peak Response			R.M.S. Response		
	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)
Uncontrolled	10·53	10·83	8·170	1·863	1·588	1·474
One PTMD installed at optimal location	6·726 (−36%)	6·874 (−37%)	4·774 (−42%)	1·237 (−32%)	0·920 (−42%)	0·743 (−50%)
Two PTMDs installed at optimal location	5·455 (−48%)	5·306 (−51%)	5·103 (−38%)	0·962 (−48%)	0·830 (−48%)	0·743 (−50%)

6.2 *Square building with large eccentricity (B2)*

As with B1, the first PTMD tuned mainly to the first mode was installed in the y direction on the top floor to control the y response on the same floor. The optimal PTMD system parameters and locations are listed in Table 10. Figure 13 shows the top floor displacement transfer functions in three directions for earthquake excitation from $\beta = 0°$ and 90°. It is apparent that the first modal amplitude in all three directional responses was suppressed significantly. Figure 14 shows the variations in mean square displacement response of the top floor with various earthquake incident angles β with and without a PTMD. It is evident that all responses were reduced for an earthquake from any incident angle.

The optimal locations and system parameters for the two PTMDs for B2, in which the second PTMD was installed in the x direction on the top floor and tuned mainly to the second mode to control the x-directional response, are listed in Table 11. From Figure 14, the response control efficacy is almost the same for both cases of one PTMD and two PTMDs. It is concluded that one PTMD is adequate for reducing both translation and rotation for a square building with large eccentricity under earthquakes from any incident angle. However, the two PTMDs case still has a slightly better response control performance.

A response control effectiveness study was also performed for B2 under the 1979 El Centro bidirectional earthquake ground accelerations. Figure 15 shows the time history displacement responses on the top floor with one, two and without a PTMD under bidirectional earthquakes. The peak and root-mean-square floor displacements in three directions are summarized in Table 12. Both peak and root-mean-square responses were reduced up to 32% when using one PTMD and 38% when using two PTMDs, as shown in Table 12.

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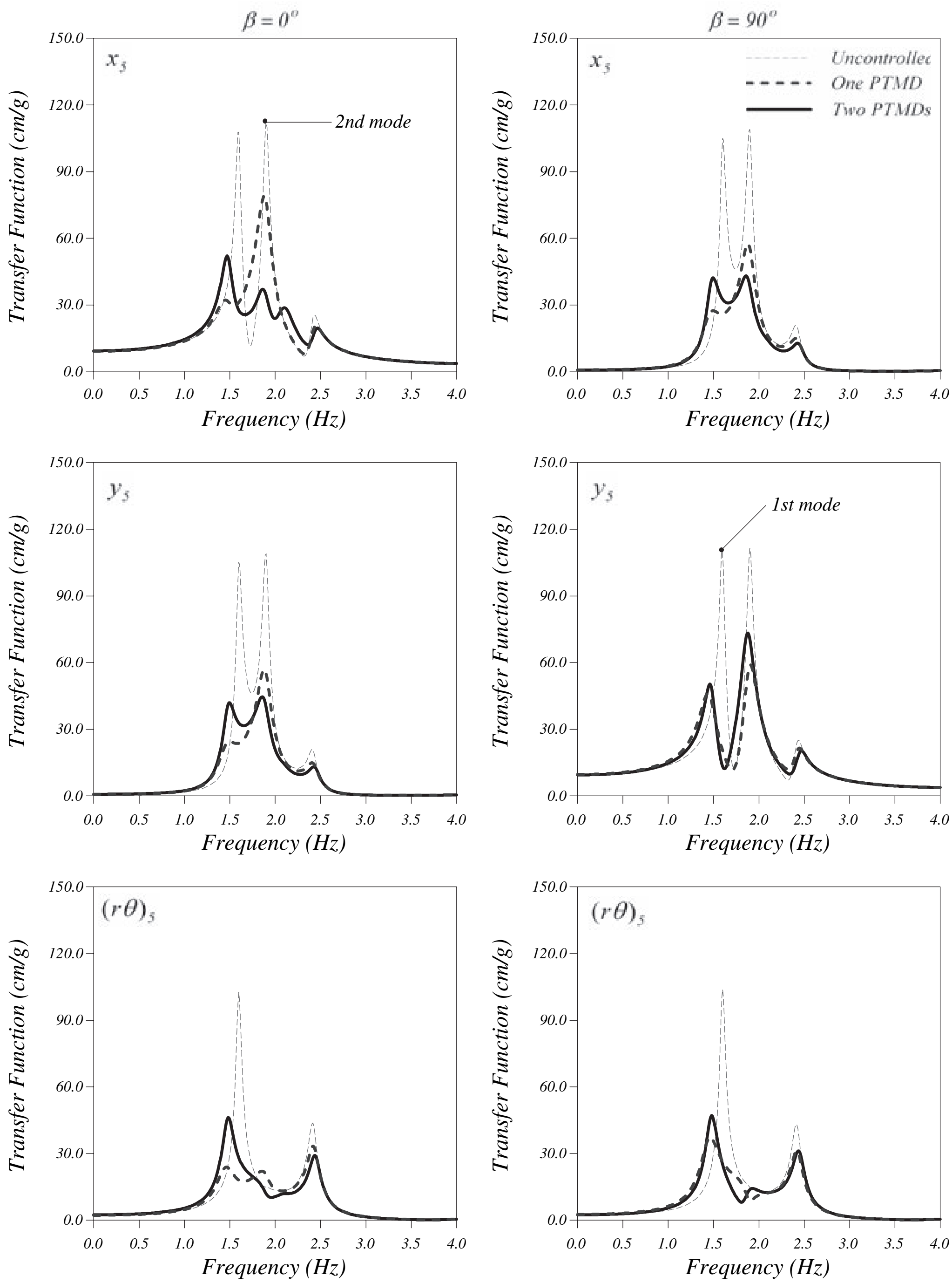

Figure 13. Top floor displacement transfer function for B2

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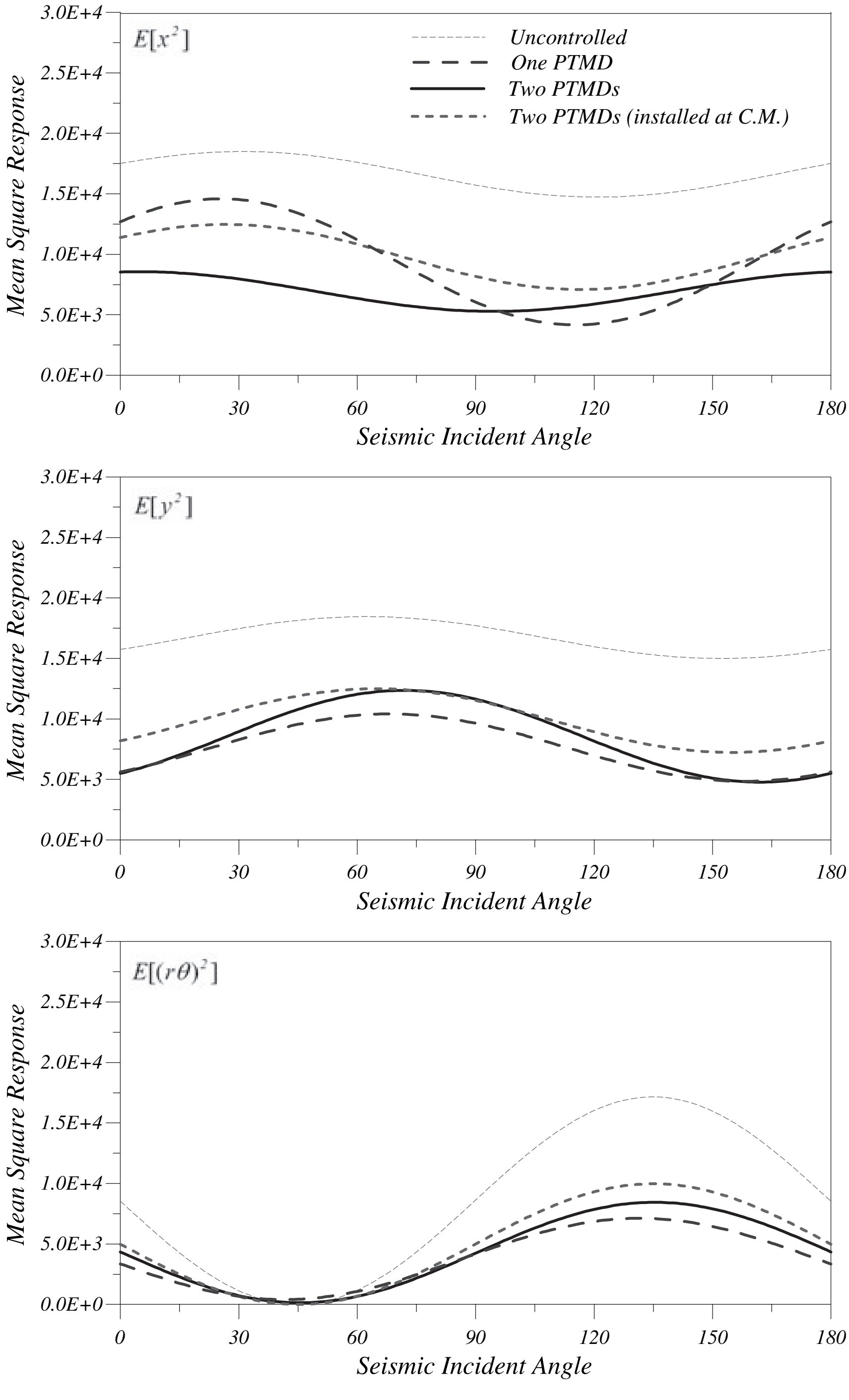

Figure 14. Top floor mean-square displacement response of B2 with and without PTMD

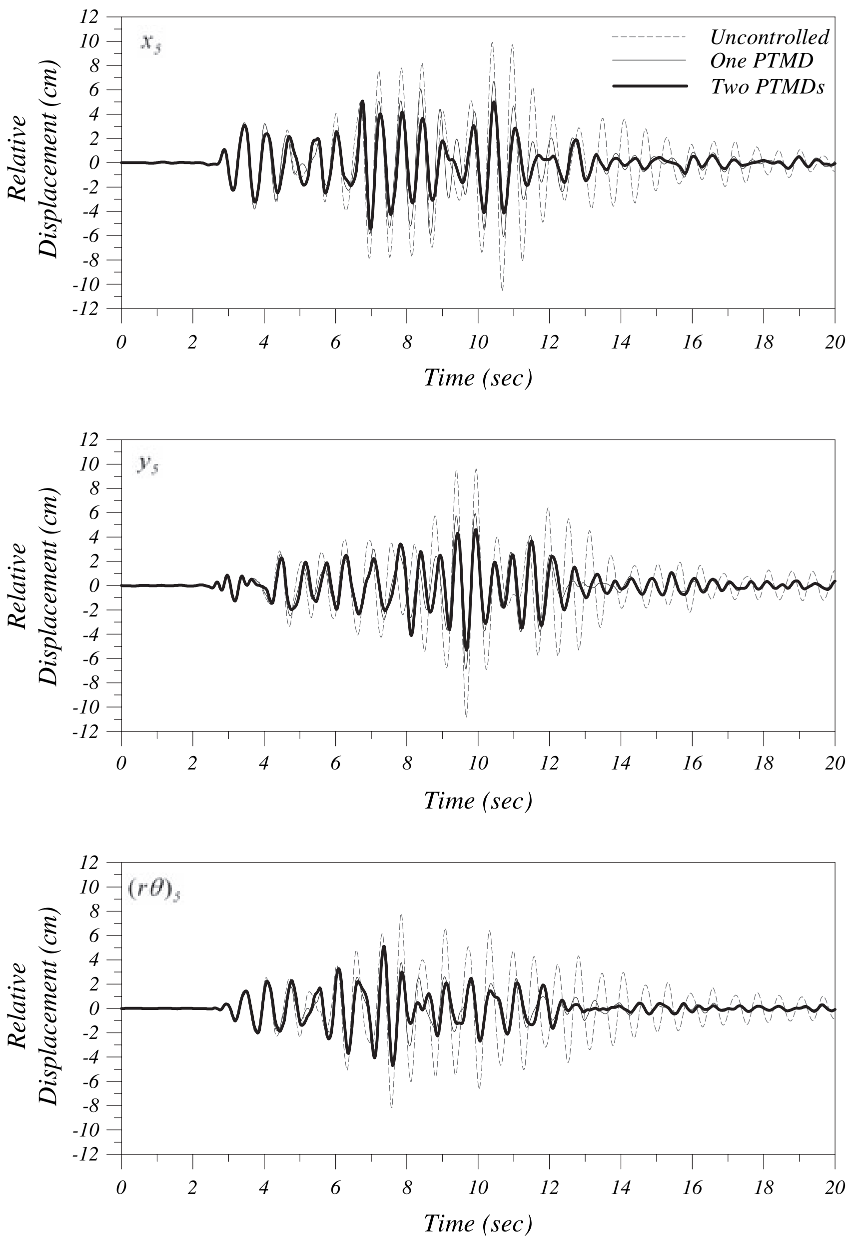

Figure 15. Top floor displacement response of B2 under bidirectional 1979 El Centro earthquakes

Figure 16 shows the detuning effect on the displacement response ratio of the top floor of B2 under the bidirectional ground accelerations of 1979 El Centro earthquakes. It was found that for this type of square building with large eccentricity the response control efficacy showed no significant variations when the PTMD parameters shifted away from the respective optimum values.

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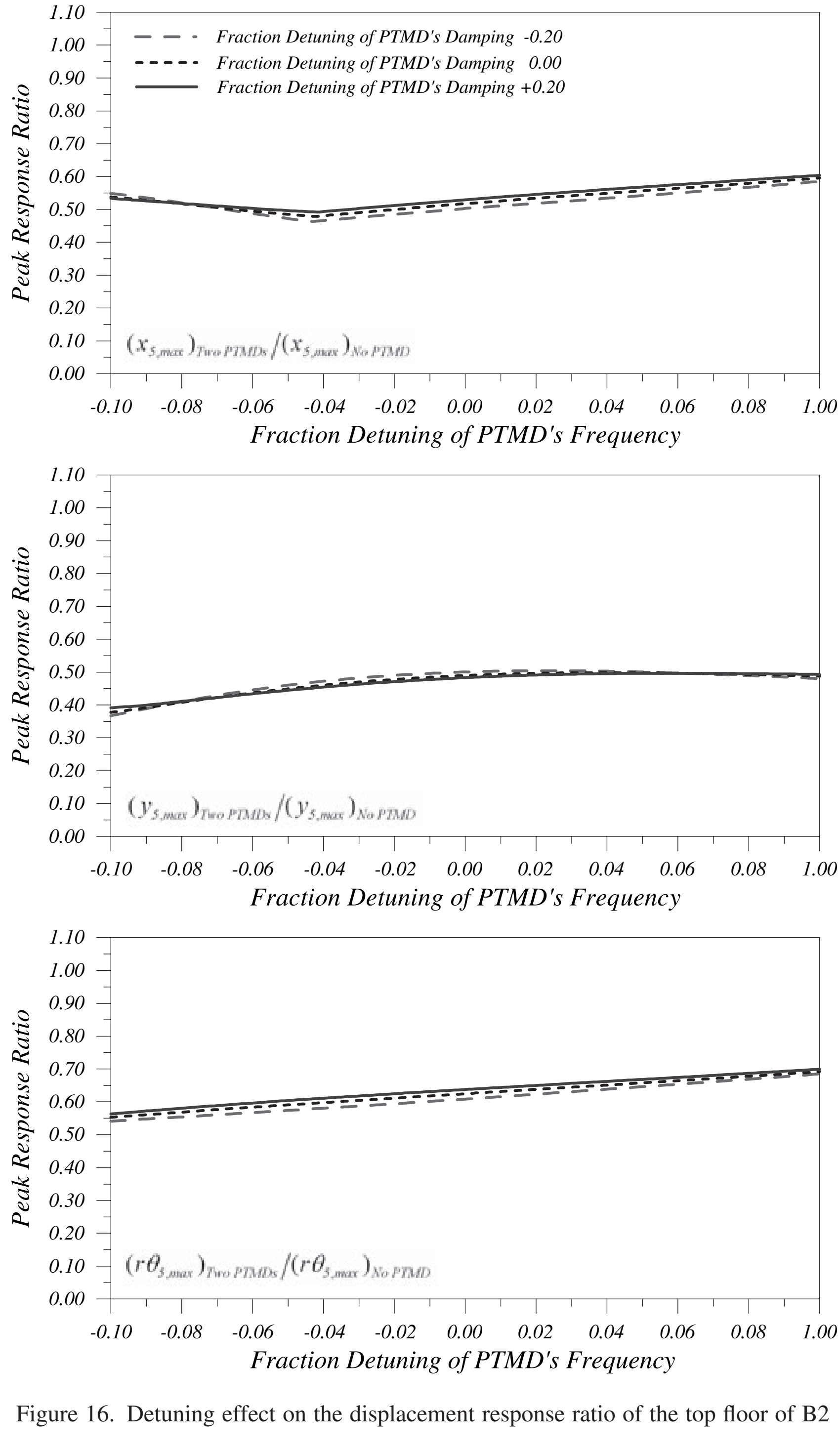

Figure 16. Detuning effect on the displacement response ratio of the top floor of B2

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7. CONCLUSIONS

The vibration control effectiveness of PTMDs for reducing the seismic responses of torsionally coupled buildings was investigated in this study. Some practical design issues such as the optimal location for the installation, planar position and movement direction of PTMDs were considered. The optimal PTMD system parameters were determined by minimizing the mean square displacement response ratio on the top floor of buildings with and without PTMDs. In addition to the vibration control effectiveness of the PTMD, parametric studies on PTMD planar positions and detuning effects for PTMD frequency and damping ratio were investigated to determine their influence on the response control efficacy. Numerical results from two typical multistory torsionally coupled buildings under bidirectional components from the 1979 El Centro earthquake were used to verify the effectiveness of the proposed optimal PTMD in reducing the building responses.

The following conclusions were drawn from these numerical examples:

(1) The PTMD is optimally installed on the top floor, to move in the same direction as the controlled DOF.
(2) The planar PTMD positions are optimally determined based on the mode shapes of the controlled mode. The greater the distance between the PTMD and mass center of the installed floor, the greater the vibration reduction obtained. A much more effective control was obtained for a PTMD installed at the optimal planar position than for one at the center of the floor mass, especially for highly coupled buildings.
(3) The response control for two PTMDs installed in two orthogonal directions produced a better performance than one PTMD installed in a weaker direction when both cases had the same total PTMD mass. However, in the case of square buildings with highly coupled DOFs, one PTMD is adequate for reducing both translations and rotation. Because square buildings have nearly the same stiffness in the *x* and *y* directions, the need for only one PTMD to be installed in the *y* direction is in conflict with general practical engineering considerations.
(4) For buildings with low-coupled DOFs, a significant variation in response control exists when the PTMD parameters shift away from the respective optimum values. Conversely, less effect was found for highly coupled buildings.
(5) Through the numerical verifications for typical buildings under bidirectional earthquake excitations, it was verified that the proposed optimal PTMDs were able to effectively reduce the building responses.

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